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Optimal routing strategy based on the minimum information path

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ABSTRACT

In this paper, we propose a new routing strategy based on the minimum information path, named the optimal routing (OR) strategy, to improve the transportation capacity of scale-free networks. We define the average routing centrality degree of the node to analyze the traffic load on nodes of different degree. We analyze the transportation capacity by using the critical values of R_c, the average packet travel time, and the average path length. Both theoretical and experimental results show that the capacity of the network under our strategy will be maximized when the packet-delivery rate of the node is directly proportional to the degree.

1. Introduction

Due to the seminal work on the small-world phenomenon [1] and the scale-free property [2], the structure and dynamics of complex networks have attracted much attention from physical and engineering communities. It has been widely proved that the topological features such as the topology and degree distribution of networks have profound effects on the processes taking place on these networks, including traffic flow.

Among all the transport problems investigated in various kinds of complex networks, information transport in communication networks such as the Internet and urban networks may be of practical importance, as a result of the increasing importance of large communication networks. In 1996, the traffic jamming transition in the Internet was first observed and analyzed [3]. It has been found that the optimal performance of transportation depends strongly on both structural characteristics of the underlying network and the routing algorithm of traffic. Compared with the high cost of changing the infrastructure of a network, developing a better routing strategy is usually preferable to enhance the network capacity. Recently, a series of heuristic algorithms have focused on developing better packet routing strategies to enhance traffic flow and to avoid traffic congestion on a large growing communication network. In those works, packets mainly are forwarded according to the following strategies: the random walking (RW) strategy [4,5], the shortest path (SP) strategy [6], the efficient path (EP) strategy [7,8], the local information (LI) strategy [9–13], and also optimized methods of the above four basic strategies [14–20]. Among them, the SP strategy is widely used in different systems. However, it often leads to the failure of hub routers with high degree and betweenness. Several optimized methods have been proposed to improve the transport capacity of the SP strategy, by means of minimizing the highest betweenness. In the EP strategy, the path between any nodes i (source) and j (destination) is defined as the path where the sum of the degrees of the nodes is a minimum. It is denoted as $p_{ij} = \min \sum_{n=0}^{l} k_n^{\beta}$, where k_n is the degree of node n, l is the path length, and β is a tunable parameter. When $\beta = 0$, the EP strategy is equivalent to the *SP* strategy. When $\beta = 1$, the EP strategy can achieve a very high capacity of the network. In the LI strategy, the packets are forwarded by the local information of the neighbors' degree, $\prod_{l\to i} = k_i^{\alpha} / \sum_j k_j^{\alpha}$. When $\alpha = 1$, the packets are forced to select low-degree nodes. The capacity of the network will be maximized when all nodes have the same packet-delivery rates.

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It should be noticed that the routing strategies mentioned above assume a constant (degree-independent) packet generation rate λ and a constant rate of delivering one packet per time step at each node. However, the nodes in a complex network such as the Internet may represent very different entities. For example, some nodes may just be individuals and others may represent big companies or universities. Obviously, different nodes will have different rates of creating messages. The nodes, depending on their connectivity to other nodes and perhaps hardware, also have different rates of delivering messages. Recently, several studies have presented a more realistic model of communication in complex networks, which includes node-dependent creation and delivering rates. Systems with packet generation rate of the form λk_n and delivering rate $1 + \beta k_n$ [21,22] have been proposed and studied, where λ and β are constants. In [23,24], the studies consider the effects of time-varying packet generation rates in the performance of communication networks, and propose a self-adjusting routing strategy in scale-free networks.

In [25–30], the authors investigate how different network topologies influence the average amount of information that is needed to send a signal from one node to another node in the network. They utilize the "search information" S to characterize the ease or difficulty of navigation in different networks. A large S means that you have to know a lot of information to find your path around in a network as a newcomer. The path with minimum information is also the path between two nodes which is first found by a random walker. Recently, we studied the probability of searching for a fixed path $\varepsilon_0\varepsilon_1\cdots\varepsilon_l$ on a network through random walks. We analyzed the first hitting time of tracking the path, and obtained an exact expression of the mean first hitting time $\langle T\rangle$. We found that $\langle T\rangle$ is divided into two distinct parts and that the first part $2m\prod_{n=1}^{l-1}k_n + \Phi$ is related to the path itself and is proportional to the degree product [31,32].

With the original contributions proposed both in [25–30] and in [31,32], in this paper, we propose a new routing strategy, named the optimal routing (OR) strategy, based on the minimum information path, to improve the transportation capacity of scale-free networks. We define the average routing centrality degree of the node to analyze the traffic load on nodes of different degree. We analyze the transportation capacity by using the critical values of R_c, the average packet travel time, and the average path length. Both theoretical and experimental results show that the capacity of the network under our strategy will be maximized when the packet-delivery rate of the node is directly proportional to the degree.

2. The "search information"

To characterize the ease or difficulty of navigation in different networks, [25–30] use the "search information", S. For any path $p(i \to j)$ from i to j, we calculate the probability $P\{p(i \to j)\}$ of following the path if one without prior knowledge would choose any new direction with equal probability:

$$P\{p(i \to j)\} = \frac{1}{k_i}\prod_{n\in p(i\to j)}\frac{1}{k_n - 1}, \tag{1}$$

with n counting all nodes on the path from a node i to the last node before the target node j is reached. The factor $k_n - 1$ instead of k_n takes into account the information we gain by following the path, and therefore reduces the number of existing links by one. The total information needed to identify one of all the degenerate paths between i and j defines the "search information". Thus, the total probability of locating node j along any of the degenerate shortest paths is [25–30]

$$S(i \to j) = -\log_2\left(\sum_{p(i\to j)} P\{p(i \to j)\}\right), \tag{2}$$

where the sum runs over all degenerate paths that connect i with j. A large $S(i \to j)$ means that one needs many yes/no questions to locate j. The existence of many degenerate paths will be reflected in a small S and consequently in easy goal finding. The average search information S_{ave} is defined to estimate the total information needed to construct the total fixed routing table:

$$S_{ave} = \frac{1}{N^2}\sum_i\sum_j S(i \to j), \tag{3}$$

where N is the size of the network.

The path with minimum information is also the path between two nodes which is first found by a random walker. Recently, we studied the probability of searching for a fixed path $\varepsilon_0\varepsilon_1\cdots\varepsilon_l$ on a network through random walks. Let us denote any path with length l on the network as $C(l) = c_0c_1\cdots c_l$. Let the walker start from source node s, and travel on the network randomly. Its walking path can be denoted as $W(t) = v_0v_1\cdots v_t$. If $C(l) \subseteq W(t)$, then the walker detects path $C(l)$. The probability $\theta(t)$ that the walker detects $C(l)$ in t steps and arrives at the initial node of $C(l)$ in the end can be calculated as [31,32]

$$\theta(t) = \sum_{r=0}^{t-l}(P_{su}^{(r)} - \theta(r)) \bullet P_0 \bullet P_{vu}^{(t-l-r)}, \tag{4}$$

where $P_{ij}^{(m)}$ is the m-step transition probability from v_i node to v_j node, $P_0 = P_{c_0c_1} \cdots P_{c_{l-1}c_l}$. Furthermore, the probability $\xi(t)$ that the walker passes through $C(l)$ at t step $(d = v)$ can be calculated as [25,26]

$$\xi(t) = [P_{su}^{(t-1)} - \theta(t - l)] \bullet P_0. \tag{5}$$

As shown in Eq. (11) of [32], the average time $\langle T \rangle$ which the walker takes to pass through $C(l)$ can be calculated by the generating function $\Re(x)$ as

$$\langle T \rangle = \Re'(x) = 2m \prod_{n=1}^{l-1} k_n + \Phi, \tag{6}$$

where m is the total number of nodes in the network, and the parameter Φ is determined by the structural characteristics of the underlying network. According to Eq. (6), we can find that the time which a random walker takes to find a given path is directly proportional to the continued product of the degrees of all the nodes which pass through the given path when the network is determined.

3. The optimal routing strategy based on the minimum information path

Let us consider the difference between Eq. (1) and Eq. (6). For any subsequent node b coming from a, the probability of locating the next node c is $1/(k_b - 1)$, because the incoming link is known according to Eq. (1). In Eq. (6), the probability of locating the next node c is $1/k_b$, because the node c is selected through a random walk. In this study, we select the path $p(l : i \to j)$ which enables the walker to take the minimal information by utilizing Eq. (6). The path must satisfy the following condition:

$$p_{\min}(l : i \to j) = \min \prod_{m=i}^{j} k_m. \tag{7}$$

Let us reconstruct Eq. (7) by using $\log(\bullet)$. For any path between nodes i and j as $p(i \to j) := i \equiv v_0, v_1, \ldots, v_n \equiv j$, we propose the optimal routing (OR) strategy based on the following condition:

$$p(i \to j) = \min \sum_{n=0}^{l} \log(k_n). \tag{8}$$

As shown in Eq. (8), our routing strategy is similar to the EP strategy, while the difference between the two strategies is that we select the path where the sum of $\log(k_i)$ instead of k_i is the minimum.

Let the matrix **deg** be the adjacency degree matrix of the network. If the edge $\langle v_i, v_j \rangle$ exists, $\mathbf{deg}[i, j]$ is equal to the degree k_j of the node v_j. Otherwise, $\mathbf{deg}[i, j] = \infty$. Let the set D be the set of end nodes of optimal paths which start from the original node v_n. $pdt[i]$ is the continued product of the degrees of nodes which pass through the optimal paths from v_n to v_i. By using Dijkstra's shortest path algorithm [33], our routing strategy is described as follows.

Step 1. Let D be null. $pdt[i] = k_n \bullet \mathbf{deg}[n, i]$, where $v_i \in V - \{v_n\}$, $V = \{v_i\}_{i=1}^{N}$ is the set of all nodes with size N.

Step 2. Select v_j which makes $pdt[j] = \min\{pdt[i] | v_i \in V - D - \{v_n\}\}$. Let $D = D \cup \{v_j\}$.

Step 3. For any node $v_k \in V - D - \{v_n\}$, if $pdt[j] \bullet \mathbf{deg}[j, k] < pdt[k]$, let $pdt[k] = pdt[j] \bullet \mathbf{deg}[j, k]$.

Step 4. Iterate steps 2–3 $N - 1$ times, and then we can get the optimal path from the node v_n to any other node v_j.

Step 5. Iterate steps 2–4 $N - 1$ times, and then we can get all of the optimal paths between any two nodes.

We can calculate that the time complexity from steps 2–3 is $O(N)$. According to step 4, the time complexity to get the optimal path from node v_n to any other node v_j is $O(N^2)$. As a result, the total time complexity getting all of the optimal paths between any two nodes is $O(N^3)$. Let us take a BA network with size $N = 1000$ as an example. By using a computer with 3.2 GHz Intel CPU and 2 GB memory, the time taken to find all of the optimal paths between any two nodes is only 32 s, which is similar to the time taken by using the *SP* strategy. If there are several optimal paths between two nodes, one is chosen at random. As a result, the fixed routing table is designed on the basis of optimal paths.

4. Simulation results

Recent studies indicate that many communication networks such as the Internet and the WWW are heterogeneous, with degree distribution following the power-law distribution $P(k) \sim k^{-\gamma}$. In this study, we use the well-known Barabási–Albert (BA) scale-free network model [2] as the physical infrastructure to study the information traffic flow.

Firstly, let us consider the average search information S_{ave}, which is the cost under different routing strategies. The relations between S_{ave} and the size N or the average degree $\langle k \rangle$ are shown in Figs. 1 and 2. Compared with the *SP* routing strategy and EP routing strategy, we can construct our fixed routing table with the minimal search information, which shows the better performance of our strategy.

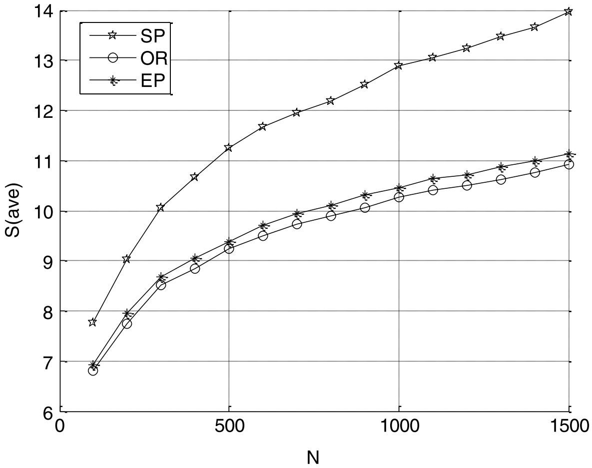

Fig. 1. The relations between the average search information S_{ave} and the size N, $m = 3$.

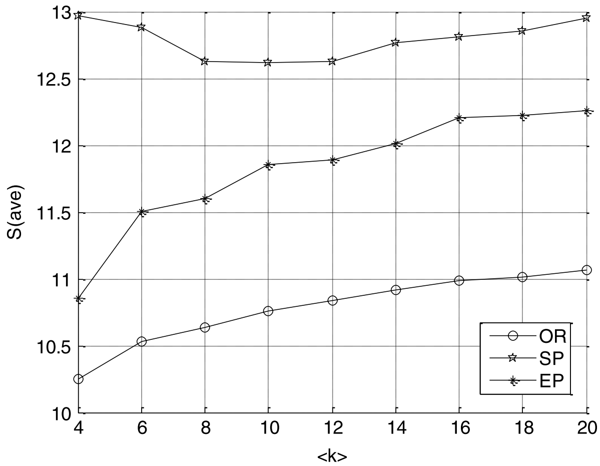

Fig. 2. The relations between the average search information S_{ave} and the average degree $\langle k\rangle$, $N = 1000$.

Similar to the definition of the betweenness centrality of node n, we define the average routing centrality degree (ARCD) of a node n in the scale-free network as

$$B(k) = \frac{1}{N(k)} \sum_{k_i=k} \sum_{s\neq t} \delta_{st}(i) \tag{9a}$$

$$\delta_{st}(i) = \begin{cases} 1 & \text{if } v_s \to v_t \text{ pass through } v_i \\ 0 & \text{otherwise,} \end{cases} \tag{9b}$$

where $N(k)$ is the total number of nodes with degree k. The ARCD gives an estimate of the traffic load on nodes of different degree. The relationships between the ARCD and degree under different routing strategies are shown in Fig. 3. As shown in Fig. 3(a), $B(k) \sim k^{1.7}$ when we select the SP routing strategy, which means that the large-degree nodes endure a much heavier traffic load than that of the low-degree nodes. Congestion occurs at the large-degree nodes even when the delivering rate of the node is directly proportional to its degree, because $B(k)/k \sim k^{0.7}$. As shown in Fig. 3(b), the ARCD distribution under the EP routing strategy is similar to a Poisson distribution. When $k \approx 20$, the ARCD reaches its maximum. Instead of the large-degree or low-degree nodes, the middle-degree nodes, whose degrees are around 20, endure a much heavier traffic load. As a result, congestion occurs at the middle-degree nodes. In contrast, the large-degree nodes are not able to make full use of their delivering capacity. As shown in Fig. 3(c), under our routing strategy, $B(k) \propto k$. The relation between the ARCD and the node degree is linear. When the delivery rate of nodes is directly proportional to the degree ($C(k) = k$), the network load is assigned properly.

It should be noticed that in a BA network the information packet can be transmitted between any node pair (i, j) in finite steps with the above three routing strategies. In order to characterize the network capacity, we use the order parameter

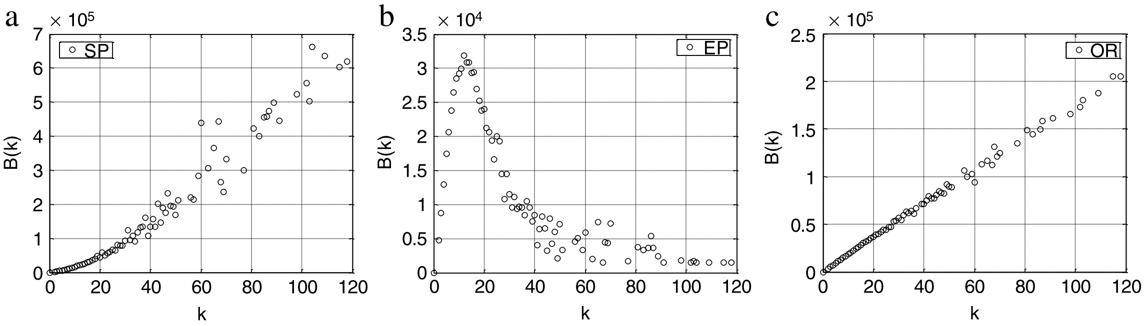

Fig. 3. The relationships between the ARCD and degree under different routing strategies: (a) SP routing strategy; (b) EP routing strategy; (c) our routing strategy.

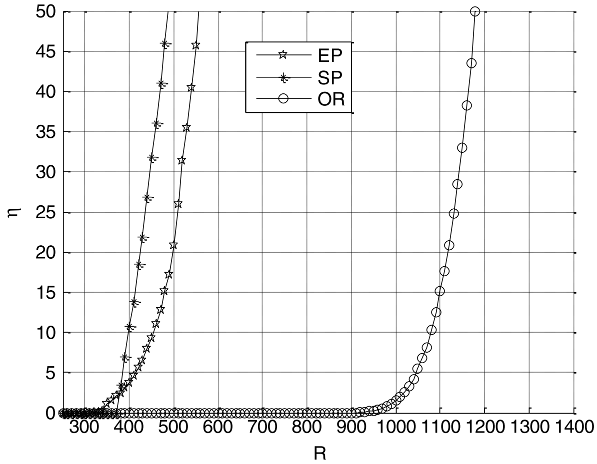

Fig. 4. The order parameter η versus the generating rate R under different routing strategies when $C(k) = k$.

presented in [34]:

$$\eta(R) = \lim_{t\to\infty} \frac{1}{R} \frac{\langle \Delta N_p \rangle}{\Delta t}, \tag{10}$$

where $\Delta N_p = N_p(t+\Delta t) - N_p(t)$, $\langle \bullet \rangle$ indicates the average over time windows of width Δt, and $N_p(t)$ represents the number of data packets within the network at time t. With increasing packet generation rate R, there will be a critical value of R_c that characterizes the traffic phase transition from free flow to a congested state. When $R < R_c$, $\langle \Delta N_p \rangle = 0$ and $\eta(R) = 0$, corresponding to the case of a free-flow state. However, for $R > R_c$ and $\eta(R)$ a constant larger than zero, the packets will continuously pile up within the network, and the system will ultimately collapse. Therefore, R_c is the maximal generating rate under which the system can maintain its normal and efficient functioning. Thus the overall capacity of the system can be measured by R_c. Let us use the BA model with $m = 2$, $\langle k \rangle = 4$, and network size $N = 1500$ fixed for simulation. Fig. 4 reports the order parameter η versus generating rate R under different routing strategies when $C(k) = k$. The critical values of R_c under different routing strategies are $R_c^{sp} = 372$, $R_c^{eff} = 315$ and $R_c^{opt} = 920$, respectively. Fig. 5 reports the critical values of R_c versus $\langle k \rangle$ under different routing strategies. Obviously, our routing strategy has better performance. We can also characterize the effectiveness of our routing strategy by calculating the average transmission time of all packets in the network, $\langle T \rangle$. Fig. 6 gives the variation of average packet travel time $\langle T \rangle$ under different routing strategies. We can get the same results of the critical values of R_c which are also reflected in Fig. 4.

With the size of the network increasing, the average routing path length will increase. Let us take a BA scale-free network with $N = 1500$, $m = 2$, and $\langle k \rangle = 4$ as an example. Under the *SP* routing strategy, the EP routing strategy, and our routing strategy, the average routing path lengths are $L_{\mathrm{sp}} = 4.16$, $L_{\mathrm{EP}} = 6.63$, and $L_{\mathrm{OP}} = 4.69$, among which our strategy ensures an average path length close to the shortest. Fig. 7 reports L_{ave} versus $\langle k \rangle$ under different routing strategies. With the increase of $\langle k \rangle$, L_{OP} gets closer to L_{sp}.

Taking both R_c and L_{ave} into account, we find that our routing strategy can achieve the maximum capacity of the network when $C(k) = k$. This phenomenon can be explained with the proper use of the large-degree nodes. Let us take a BA scale-free network as an example, whose topological structure is shown in Fig. 8. Under different routing strategies, the average routing path lengths are $L_{\mathrm{sp}} = 2.68$, $L_{\mathrm{EP}} = 3.05$ and $L_{\mathrm{OP}} = 2.76$. For any path such as $p(v_{22} \to v_{32})$, we can get three

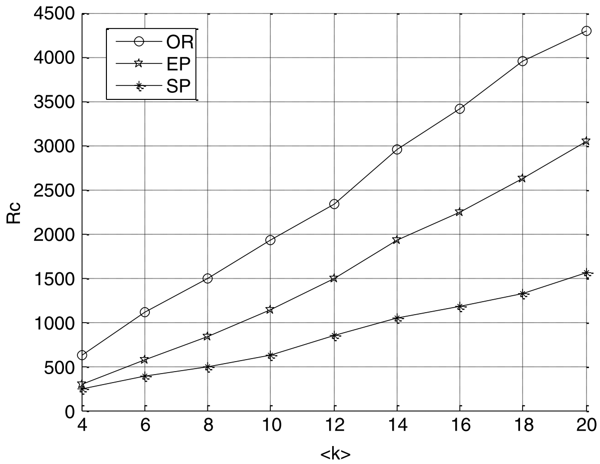

Fig. 5. The critical values of R_c versus $\langle k \rangle$ under different routing strategies with $N = 1000$.

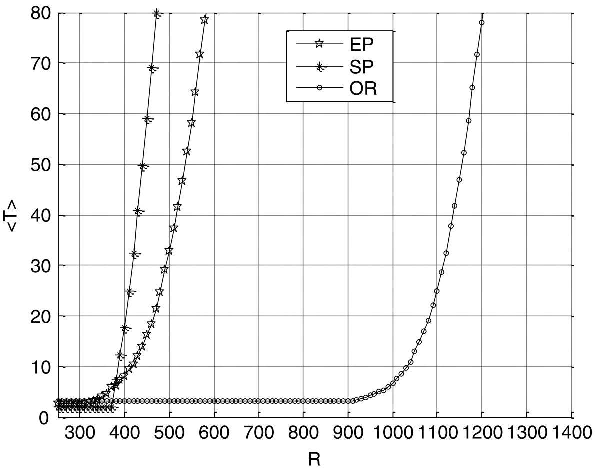

Fig. 6. The variation of average packet travel time $\langle T \rangle$ under different routing strategies.

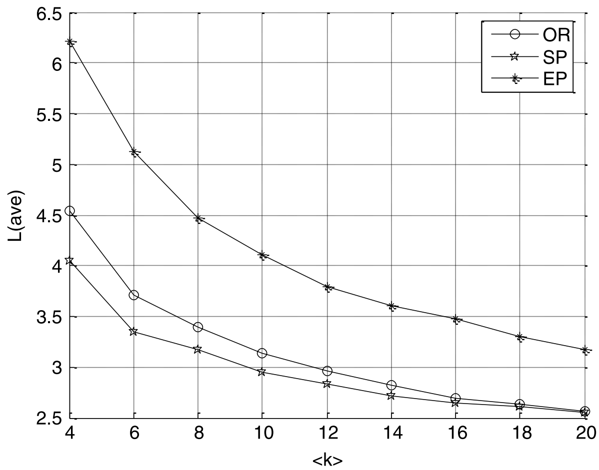

Fig. 7. The average routing path length L_{ave} versus $\langle k \rangle$ under different routing strategies with $N = 1000$.

different routing paths: $p_{sp} = \{v_{22}, v_5^{15}, v_4^{10}, v_{32}\}$, $p_{ep} = \{v_{22}, v_{31}^3, v_6^6, v_{29}^3, v_{12}^6, v_{32}\}$, and $p_{opt} = \{v_{22}, v_{13}^{16}, v_{41}^2, v_{38}^3, v_{32}\}$, respectively. As shown in Fig. 8, the path passing through the white nodes is p_{opt}. The path passing through the light grey nodes is p_{sp}. The path passing through the dark grey nodes is p_{ep}. Under the SP routing strategy, the walker prefers to choose

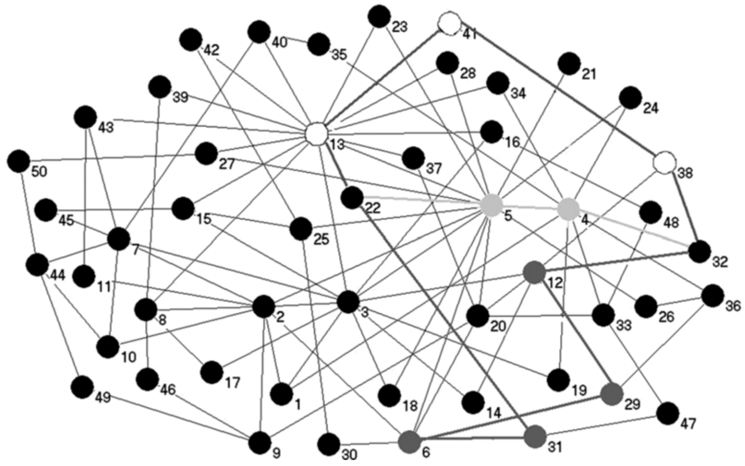

Fig. 8. A simple scale-free network generated by using the BA model with $N = 50$ and $\langle k \rangle = 4$. The path passing through the white nodes is p_{opt}. The path passing through the light grey nodes is p_{sp}. The path passing through the dark grey nodes is p_{ep}.

the large-degree nodes, to make the routing path the shortest. As a result, the large-degree nodes v_5 and v_4, whose degrees are 15 and 10, respectively, are selected. However, the preference of large-degree nodes leads to the failure of hub routers with high degree and betweenness. In contrast, under the EP routing strategy, the walker tends to select the low-degree nodes, to redistribute the heavy load from the central nodes (with the highest connectivity) to the lower degree nodes. The low-degree nodes (v_{31}, v_6, v_{29}, and v_{12}) are selected, and all the high-degree nodes are avoided. The tendency of selecting low-degree nodes makes L_{EP} much longer than L_{sp}. Different from the SP routing strategy and the EP routing strategy, our routing strategy selects the high-degree nodes in a more proper way. The routing path $P(v_{22} \to v_{32})$ under our strategy is $P_{opt} = \{v_{22}, v_{13}^{16}, v_{41}^{2}, v_{38}^{3}, v_{32}\}$. We utilize some high-degree nodes such as v_{13} to reduce the routing path length. At the same time, we avoid completely relying on the high-degree nodes, and thus distribute the load efficiently. Compared with the SP routing strategy, we optimize the load to satisfy $B(k) \propto k$ at the cost of increasing the average routing path length slightly. The compromise of our strategy is worthwhile, as the capacity of the network is maximized when $C(k) = k$.

5. Conclusion

In this paper, we propose a new routing strategy, named the optimal routing (OR) strategy based on the minimum information path, to improve the transportation capacity of scale-free networks. We define the average routing centrality degree of the node to analyze the traffic load on nodes of different degree. We analyze the transportation capacity by using the critical values of R_c, the average packet travel time, and the average path length. Both theoretical and experimental results show that the capacity of the network under our strategy will be maximized when the packet-delivery rates of the node is directly proportional to the degree.

It should be noted that the study of an optimal routing strategy based on the minimum information path is still in its primary stage, and further efforts need to be made. For example, we cannot prove why the average routing centrality degree of a node is proportion to its degree. As a result, some further investigations of this aspect need to be made.

Acknowledgements

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