# TRANSITIVITY IN STRUCTURAL MODELS OF SMALL GROUPS

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**Our purpose here is to show** how various deterministic models for the structure of interpersonal relations in small groups may all be viewed as special cases of a single model: namely, a transitive graph (t-graph). This exercise serves three purposes. First, the unified approach renders much of the mathematical discussion surrounding these various models quite transparent. Many of the arguments boil down to nothing more than defining certain equivalence relations and looking at the resulting equivalence classes. Second, our focus on the general model may stimulate the search for other useful specializations besides those indicated here. We discuss two ways of specializing the model—restrictions on edges and on triads—but other methods can be used. Third, we propose to adopt transitivity as the key structural concept in the

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analysis of sociometric data. Other models then become "transitivity plus something else." This provides a framework for the analysis of the fine structure of sociometric data.

Transitivity is not a new idea in the study of interpersonal relations. It is, for example, fundamental to Heider's balance theory. The association between transitivity and "balance" in positive interpersonal sentiment is explicit in several passages of Heider's writing. For example, he remarks that, "In the p-o-x triad, the case of three positive relations may be considered psychologically . . . transitive" (Heider, 1958: 206; compare also: Heider, 1946: 109). The formalization of Heider's theory into the model of "structural balance" by Cartwright and Harary (1956) is, in fact, a very special case of a transitive graph and does not deal with the full generality of Heider's conception as we interpret it here. While Heider was concerned with cognitive balance involving at most three entities, we are interested in the structural consequences of transitive graphs of actual interpersonal relations among many individuals.

Rapoport has used the notion of a "transitivity bias" in the analysis of sociometric data (e.g., Rapoport, 1963). His approach is a probabilistic rather than a deterministic one. Furthermore, for Rapoport, transitivity is only one of a variety of "biases" that affect the structural tendencies in a network; while we are concerned with seeing how far we can go with a deterministic transitive structure.

## DESCRIPTION OF A GENERAL TRANSITIVE GRAPH

### NOTATION

We adopt the following notation throughout the paper. The set, X, is composed of individuals x, y, z, u, v, w. . . . A binary relation, C, is defined on X. The notation, x C y, indicates that x expresses positive sentiment toward y, or, briefly, that "x chooses y." Equivalently, C defines a directed graph on the elements of X–a directed edge goes *from* x *to* y if and only if x C y. The relational and graph theoretical interpretations of C will be used interchangeably. Three other relations may be defined on X in terms of C. These are:

(1) x M y if and only if x C y *and* y C x.

(2) x A y if and only if x C y *and not* y C x.

(3) x N y if and only if *neither* x C y *nor* y C x.

The relations M, A, and N have simple sociometric interpretations: M denotes mutual choices; A denotes asymmetric or unreciprocated choices; and N denotes null or mutual non-choices. The relations M and A determine C and N so that we may take either (X, C) or (X, M, A) as the basic data and derive the other relations from them. If x M y then we say that x and y are joined by an M-edge. Similarly for A-edges and N-edges. If there are no pairs of *distinct* individuals for which x M y holds then we shall say that M is empty. The same goes for A and N. Finally, we shall have cause to refer to certain subsets of X (such as, M-cliques). These will be denoted by capital letters, U, V, W . . . , and relations defined on these subsets will be denoted by capital letters followed by a superscript asterisk, for example, $A^*$, $N^*$.

## MATHEMATICAL DEFINITION OF A TRANSITIVE GRAPH

Definition 1: (X, C) *is a transitive graph (t-graph) if and only if for all* x, y, z *in* X:

(a) x C x

(b) *if* x C y *and* y C z *then* x C z.

For us, the condition (a), reflexivity, is merely a convention to avoid trivial exceptions and no substantive concern is given to it. In the graph theoretic interpretation, condition (a) implies that there are loops at each vertex, but we always ignore these loops. Note that a t-graph is closely related to the notion of a partial order which is a t-graph for which M is empty (i.e., for no distinct elements of X does x M y hold).

## THE STRUCTURE OF A TRANSITIVE GRAPH[1]

Theorem 1 summarizes several easily proved properties of M, A, and N when (X, C) is a t-graph. Its proof is omitted.

*Theorem 1: If* (X, C) *is a t-graph, then for all* x, y, z *in* X:

(a) x M x

(b) x M y *implies that* y M x

(c) x M y *and* y M z *imply that* x M z

(d) x A y *implies that not* y A x

(e) x A y *and* y A z *imply that* x A z

(f) *not* x N x

(g) x N y *implies that* y N x.

Conditions (b), (d), and (g) do not require (X, C) to be a t-graph. Parts (a), (b), and (c) of Theorem 1 assert that when (X, C) is a t-graph, then M is an equivalence relation on X (reflexive, symmetric, and transitive). Thus M partitions X into a system of mutually exclusive and exhaustive subsets with the property that x and y are in the *same* subset if and only if x M y. To indicate that these subsets are defined by M and to give them a name consonant with their social structural interpretation we call them the M-cliques of X and note that an M-clique may be of size one. Each group member who is involved with another in a mutually positive pair relation is in an M-clique with that member and is involved in a mutually positive pair relation with every other member of that M-clique.

The notion of mutuality as identifying subgroup membership is common in the sociometric and social psychological literature. A clear theoretical statement appears in Homans (1950) and it is formally modeled in terms of mutuality in Cartwright and Harary (1956), Davis (1967), and Davis and Leinhardt (1971).

The M-cliques are *compatible* with A and N in a sense that is summarized in the next theorem.

*Theorem 2: If* (X, C) *is a t-graph and if* u *and* v *are in the same* M-*clique, then for all* x *in* X:

(a) u A x *if and only if* v A x

(b) x A u *if and only if* x A v

(c) u N x *if and only if* v N x.

*Proof:* We shall prove only the "if" part of (a) as the other arguments are similar. Suppose u M v and u A x. Then we certainly have v C u and u C x, so by transitivity we must also have v C x. Could it happen that x C v obtains also? Suppose so. Then x C v and v C u imply that x C u contrary to the assumption that u A x. Therefore v A x holds. *Q.E.D.*

The content of Theorem 2 may be described by saying that in a t-graph all individuals in an M-clique are structurally equivalent in the sense that they all stand in the same relation to any other individual in the group. Thus, if any member of an M-clique chooses an individual outside of that clique, all of his fellow members also choose that individual and if one clique member fails to choose an individual outside the clique then none of his fellow clique members chooses that individual. The main implication of Theorem 2 is that an ordering relation, A*, may be defined on the M-cliques themselves, as follows:

*Definition* 2: *If* U *and* V *are two distinct M-cliques of* X, *then define* U A* V *if and only if* u A v *for all* u *in* U *and* v *in* V. *Also set* U A* U *for all M-cliques,* U.

The ordering, A*, of the M-cliques is by definition reflexive. From Theorem 1, parts (d) and (e), it follows that

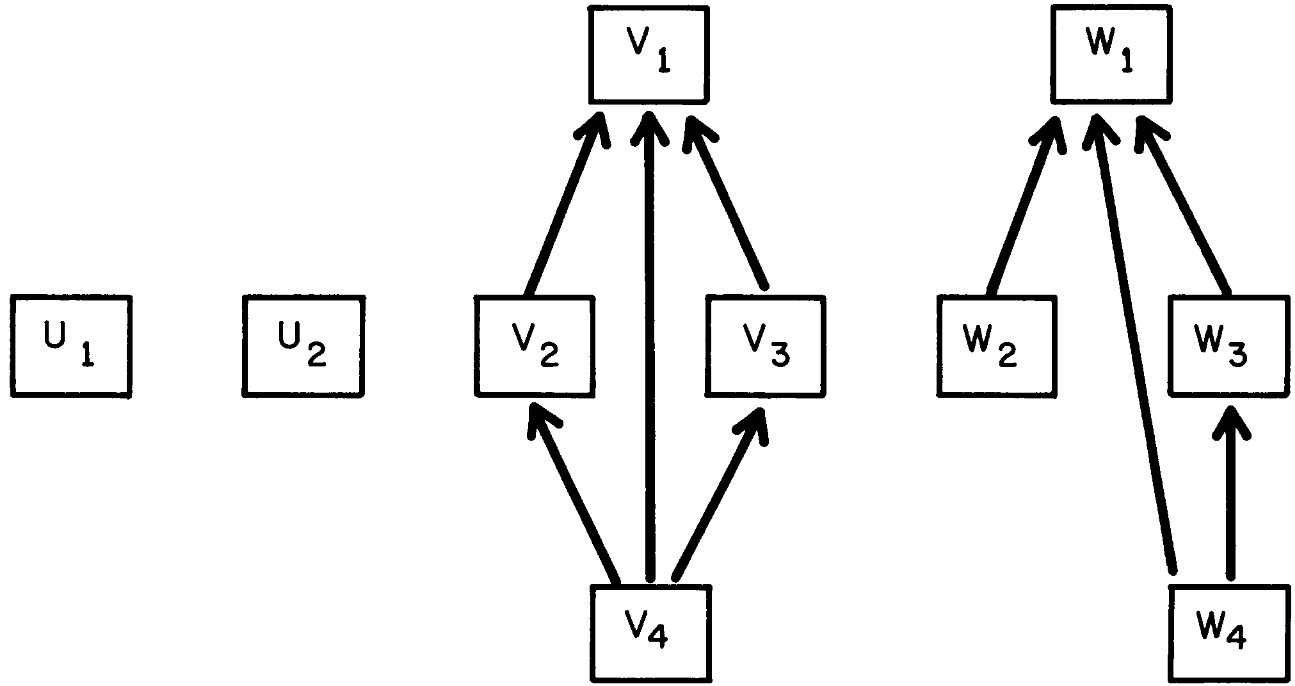


**Figure 1:** A SCHEMATIC REPRESENTATION OF ONE POSSIBLE FORM OF A T-GRAPH. THE BOXES REPRESENT M-CLIQUES AND THE ARROWS REPRESENT THE RELATION A*.

A* is also anti-symmetric and transitive. Therefore, A* is a partial order on the M-cliques of X. The next theorem summarizes the above discussion in terms of the structure of a t-graph. Its proof is evident from the previous discussion and is omitted.

*Theorem 3: If* (X, C) *is a t-graph then the elements of* X *may be partitioned into M-cliques with the following properties:*

(a) *within each M-clique all pairs of individuals are joined by M-edges;*

(b) *between any two distinct M-cliques all pairs of individuals are either all joined by A-edges with the same direction or all joined by N-edges;*

(c) *the M-cliques when ordered by* A* *form a partial order.*

A rather less intuitive statement of Theorem 3 may be found in Ore (1962: 151). Each part of Theorem 3 has a simple sociometric interpretation. In part (a) the internal structure of each M-clique is characterized by mutual positive sentiment between each pair of clique members. In part (b) the relations between pairs of M-cliques is characterized either by a status ranking of one clique over the other (the case of A-edges) or by no status ranking (the case of N-edges). Finally in part (c) the entire system of M-cliques is characterized as forming a consistent structure in the sense of a partial ordering.

## AN EXAMPLE OF A TRANSITIVE GRAPH

The class of distinct t-graphs is very large and there is little hope of accurately describing all of the possibilities in the general case. However, in the following example we try to illustrate the variety that is available with simple structures. Figure 1 illustrates a single group that contains ten M-cliques: $U_1$, $U_2$, $V_1$, $V_2$, $V_3$, $V_4$, $W_1$, $W_2$, $W_3$, $W_4$. The M-cliques may be of varying sizes, of course, but no group member may belong to more than one. Cliques $U_1$ and $U_2$ are unranked relative to any of the other cliques. The cliques $V_1$, $V_2$, $V_3$, and $V_4$ form a system of "levels" such that everyone on a lower "level" chooses everyone above him. The cliques $W_1$, $W_2$, $W_3$, and $W_4$ are involved in a ranking system but they do not form such clear-cut "levels" since the individuals in $W_4$ do not choose those in $W_2$.

Figure 1 provides an illustration of the variety of organizational patterns which may result after the conditions for a t-graph have been satisfied. It indicates as well how transitivity in positive interpersonal sentiment can result in stratification as well as clustering. This point is particularly important because of the tendency of theorists to view stratification and clustering as two separate dimensions of social structure (see, for example, Homans, 1950; Brown, 1965). While status relationships have previously been identified with asymmetry and clustering with mutuality in positive interpersonal sentiment (Davis and Leinhardt, 1971), the t-graph model indicates that these are simply different expressions of a single social organizational principle. Furthermore, if a t-graph is considered to be a generalization of Heider's balance theory, then the figure illustrates how balance leads to the development of hierarchies as well as cliques.

## SPECIAL CASES OF T-GRAPHS OBTAINED BY RESTRICTING THE TYPES OF EDGES AND TRIADS

This section is devoted to examining a variety of special cases of t-graphs. Thus all of the structures mentioned in this section are examples of t-graphs that satisfy further conditions beyond transitivity. We examine only two possible classes of restrictions one might impose on a t-graph. The first concerns restrictions on the types of dyads (or edges) that can appear in the t-graph. The second concerns restrictions on the types of triads that can appear. Other types of t-graphs are mentioned briefly in the section "Other Types of T-Graphs" (p. 121).

## RESTRICTIONS ON THE EDGES

If a t-graph has only one type of edge (i.e., all dyads are the same type), then the form of the t-graph is completely determined. "M-edges only" implies that the t-graph consists of a single completely connected M-clique. "A-edges only" implies that the t-graph forms a transitive tournament, that is, a complete linear ordering of all individuals (compare Landau, 1951-1953). "N-edges only" ensures that the t-graph is a totally disconnected set of points.

More interesting possibilities arise when the t-graph is allowed to possess edges of two but not all three types. For example, if only M- and A-edges are allowed, then Theorem 3 implies that the t-graph forms a quasi-series (Hempel, 1952). This type of structure consists of a linearly ordered series of M-cliques as indicated in Figure 2. The possibilities that may arise when only M- and N-edges are allowed are summarized in the following corollary of Theorem 3:

*Corollary 1: If* (X, C) *is a t-graph and* A *is empty, then* X *may be partitioned into M-cliques such that:*

(a) *within each M-clique, all individuals are joined by M-edges,*

(b) *between any two distinct M-cliques all individuals are joined by N-edges.*

Corollary 1 describes Davis' "clusterable graph" (Davis, 1967). This model as well as the structural balance model of Cartwright and Harary (1956) is usually stated in terms of signed graphs rather than digraphs as done here. However, if positive edges are identified with M, and negative edges identified with N, a signed graph may be considered as a special case of a digraph for which A is empty. This

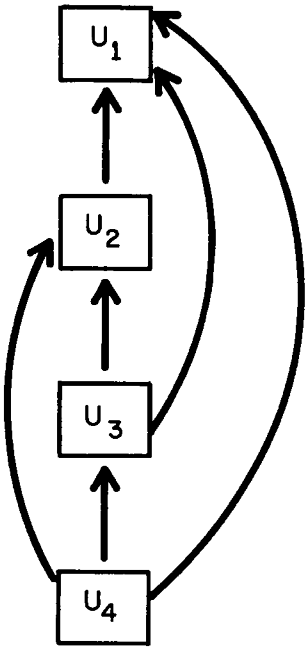


Figure 2: SCHEMATIC REPRESENTATION OF A QUASI-SERIES. BOXES REPRESENT M-CLIQUES AND THE ARROWS DENOTE THE RELATION A*.

interpretation of a signed graph is a purely conceptual device and should not be confused with the practical question of turning sociometric data—which are usually directed graphs—into signed graphs. (In this situation, M-edges are usually scored positive, N-edges are usually negative, but A-edges may be scored either positive or negative depending on one's point of view.)

As indicated in the section "Mathematical Definition of a Transitive Graph" (p. 109), no essential simplification of the structure of a t-graph occurs if no M-edges are allowed. The resulting structure may be a fully general partial order. Table 1 summarizes the result of restricting the edges of a t-graph.

**TABLE 1**

| Edge Types | Resulting Graphs |
|---|---|
| M only | Completely connected graph |
| A only | Transitive tournament |
| N only | Completely disconnected graph |
| M, A only | Quasi-series |
| M, N only | Clusterable graph |
| A, N only | Partial order |

Graph types resulting from restricting the edges of a t-graph.

## RESTRICTIONS ON THE TRIADS

Figure 3 illustrates the sixteen essentially different (non-isomorphic) triad configurations that may obtain in a directed binary graph. The triads are labeled in the manner used in Holland and Leinhardt (1970). The three numbers refer to the quantity of M, A, and N-edges in the triad, respectively. The letters U, D, T, and C further distinguish the triad types. A few technical points are worth mentioning. First, a *triad* is intransitive if for at least one of the six possible ordered triples of the individuals that make it up, say (x, y, z), it occurs that x C y and y C z but not x C z. Thus a triad may be intransitive from one, two, or all three or the individuals' points of view. Second, a triad may contain up to six transitive ordered triples. Third, if any one of the following three configurations obtains,

x C y and not y C z or

not x C y but y C z or

neither x C y nor y C z,

then the transitivity condition says nothing regarding x C z. These configurations are called vacuously transitive. In Figure 3, the transitive and vacuously transitive triads appear on the left while the intransitive triads appear on the right. The binary relation, C, will be transitive if and only if the graph

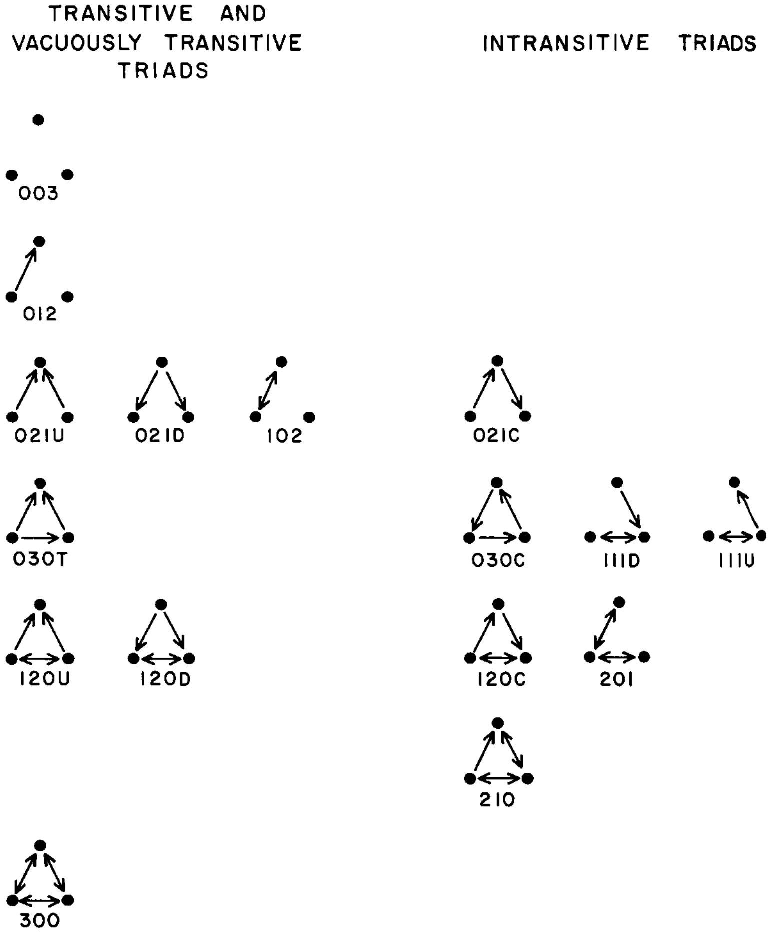


**Figure 3:** ALL 16 TRIAD TYPES ARRANGED VERTICALLY BY NUMBER OF CHOICES MADE AND DIVIDED HORIZONTALLY INTO THOSE WITH NO INTRANSITIVITIES AND THOSE WITH AT LEAST ONE.

contains no intransitive triads. Thus a t-graph is characterized by possessing only those triad types that fall on the left side of Figure 3. It is evident that restricting the allowable dyads also restricts the allowable triads—for example, when A is empty the only possible triads that any graph (transitive or not) may possess are the types 300, 201, 102, and 003.

The next model considered is structural balance. Davis' characterization of clusterable graphs is that no triad of the 201-type exists in the graph. From our point of view this characterization is equivalent to assuming that C is transitive and A is empty. The structure theorem of Cartwright and Harary (1956) states that a signed graph is balanced (separates into two cliques) if and only if there are no negative 3-cycles. This means no triads of type 201 or 003. From our point of view, structural balance assumes that A is empty and C is transitive plus an added condition that has the effect of restricting the number of possible M-cliques to two at most.

A restriction on the triad types that has the effect of limiting the number of M-cliques seems difficult to justify and this may account for the difficulty of empirically verifying the dichotomization of groups predicted by structural balance. A corresponding restriction can be found for the case when N rather than A is empty. As we have seen when N is empty, the t-graph must be a quasi-series. It will be restricted to a quasi-series with only two levels if the triad type 030T is prohibited.

The final structure we discuss is the model proposed by Davis and Leinhardt (1971). Generalizing a system of completely ranked individuals (transitive tournament) to a system of ranked cliques with one clique per level (quasi-series) suggests the further generalization to ranked *clusters* of cliques—possibly more than one clique per level. This is the model described by Davis and Leinhardt. They called it a partial order but we shall refer to it as a system of ranked clusters. Although the conditions for this model are remarkably close to the definition of transitivity, it does not reach the full generality of a t-graph. Ranked clusters allow M, A, and N all to be non-empty, unlike any of the special cases described so far. M-edges exist between members of an

M-clique as usual; N-edges exist between the M-cliques on a given level, while A-edges exist between M-cliques on different levels. Note that N-edges in the ranked clusters model link cliques of equal status while in the general t-graph cliques linked by such edges are not necessarily comparable. This difference in interpretation of the role N-edges play is crucial. Davis and Leinhardt give a characterization of their model in terms of its triads, and their interpretation of N-edges leads them to exclude a triad included in a general t-graph. In particular, they prove that a directed graph may be arranged into a system of ranked clusters of cliques if and only if it does not contain any of the intransitive triads (the entire right side of Figure 3) or any 012 triads. From our point of view, their theorem may be stated as follows:

*Theorem 4:* (Davis and Leinhardt, 1971) *A directed graph* (X, C), *is a system of ranked clusters if and only if it is a t-graph and it has no 012 triads.*

The proof of Theorem 4 given by Davis and Leinhardt uses the cluster theorem of Davis (1967). Another line of proof assumes that (X, C) is a t-graph, and examines the implication of no 012 triads for that structure. The following definition and lemma elucidate the essential logic of the argument:

*Definition* 3: *If* U *and* V *are two distinct M-cliques of* X, *then define* U N* V *if and only if* u N v *for all* u *in* U *and* v *in* V. *Also set* U N* U *for all M-cliques,* U.

*Lemma* 1: *Let* (X, C) *be a t-graph and suppose it has no* 012 *triads, then* N* *is an equivalence relation on the M-cliques.*

*Proof:* By definition N* is reflexive; also Theorem 1 (g) implies that N* is symmetric. It remains to show that N* is transitive. Suppose U N* V and V N* W for three M-cliques. Since U and W and M-cliques by Theorem 3 they either have all A-edges or all N-edges between them. Suppose they have an A-edge; e.g., u A w for some u in U and w in W. Then we have v in V such that u N v, v N w, and u A w. Hence, the triad u, v, w is of type 012 contrary to hypothesis. The same argument applies if w A u holds. Thus U and W have all N-edges between them so that U N* W. *Q.E.D.*

The argument to establish the "if" part of Theorem 4 proceeds by using N* to partition the M-cliques into mutually exclusive and exhaustive *clusters of cliques. Within* a cluster of cliques all of the cliques are joined by N-edges and *between* the clusters all of the cliques are joined by A-edges. Theorem 3 ensures that the A-edges from all the cliques in a given cluster go in the same direction so that the clusters are ranked from lowest to highest. The "only if" part of Theorem 4 follows a straightforward enumeration of the possible triads.

From the point of view of a t-graph the Davis-Leinhardt model of ranked clusters consists of an additional assumption that has the effect of drastically reducing the number of possible structures. In particular, a t-graph may contain disconnected components, but a system of ranked clusters cannot. The often noted "sex cleavage" (Gronlund, 1959; Leinhardt, 1968; Moreno, 1953) of children's classroom groups would be an allowable situation under the t-graph model but would disconfirm a model which prohibited the occurrence of 012 triads.

## OTHER TYPES OF T-GRAPHS

The restrictions we have discussed so far are all of the form "no subgraphs of size x of the type y." Our attention has been focused on subgraphs of size 2 and 3 (dyads and triads). Many of the important classes of graphs cannot be described in these terms. For example, the condition that a graph be connected cannot be described by saying that it does not possess any subgraphs of a fixed size in particular isomorphism classes. Semilattices, t-graphs proposed by Friedell (1967) as models of organizations, cannot contain disconnected components, but can contain 012 triads unlike the

Davis-Leinhardt model. Semilattices are examples of structures that cannot be characterized by the nonexistence of particular subgraph types.

## EMPIRICAL SUPPORT FOR THE T-GRAPH MODEL

Recently, Davis (1970) has carried out an analysis of 742 sociomatrices from diverse small groups. His data provide tentative empirical support for the t-graph model. For each triad type, Davis computes the percent of sociomatrices in his data bank for which the observed number of triads of the given type exceeds the number expected by chance given the distribution of M, A, and N edges in the sociomatrix. (Formulae for these expected values appear in Holland and Leinhardt, 1970.) He then uses these thirteen[2] proportions as the dependent variables to be explained by models of small group structure. The overall results of Davis' analysis appear in Table 2. We have made several modifications including the change to our triad nomenclature and the addition of three new columns:

> (1) the column headed "transitivities" contains the number of ordered triples (or points of view) in each triad type for which the transitivity condition holds and is nonvacuous; (2) the column headed "intransitivities" records the number of times in each triad that a contradiction to transitivity exists; (3) the last column contains the difference between these two quantities.

These columns aid in interpreting Davis' results, for they make it clear that the more intransitive the triad, the less frequently it appears. The columns also indicate an apparent counterbalancing effect resulting from the number of transitivities in a triad. The difference between the number of transitivities and the number of intransitivities predicts very nearly the observed order of infrequency. Those triads with more transitivities are more frequent than chance expectation, and those with more intransitivities are less frequent than chance expectation in proportion to the difference in

TABLE 2
DEFICITS OF TRIADS IN 742 SOCIOMATRICES

| Triad Type | Percent Matrices with Triad Infrequent[a] | Transi-tivities | Intransi-tivities | Transitivities minus Intransitivities |
|---|---|---|---|---|
| 030C | 90 (441)[b] | 0 | 3 | -3 |
| 201 | 90 (651) | 0 | 2 | -2 |
| 021C | 83 (701) | 0 | 1 | -1 |
| 111U, 111D | 78 (716) | 0 | 1 | -1 |
| 120C | 75 (563) | 1 | 2 | -1 |
| 012 | 42 (708) | 0 | 0 | 0 |
| 003 | 41 (649) | 0 | 0 | 0 |
| 210 | 37 (530) | 3 | 1 | 2 |
| 102 | 37 (706) | 0 | 0 | 0 |
| 021U, 021D | 27 (701) | 0 | 0 | 0 |
| 030T | 16 (601) | 1 | 0 | 1 |
| 120U, 120D | 7 (567) | 2 | 0 | 2 |
| 300 | 1 (301) | 6 | 0 | 6 |

a. From: Davis, 1970, Table 1, "Total" column: "Percent of Matrices with Triad Frequency Less Than Chance Expectation."

b. Number of matrices with expected value of 1.00 or greater. Triad predictions were not tested when expectation of a triad fell below 1.00 in a matrix.

these two characteristics. The only triad-type that violates this proposition is the 210, but we hasten to point out that our choice of the *difference* between transitivities and intransitivities was arbitrary and that it might be better to give the one intransitivity of the 210 triad more weight than we have.

One substantive interpretation of these results is that there is a tendency in sociometric data away from imbalance (i.e., intransitivity) and that when imbalance does occur it is resolved through transitive closure rather than through the development of vacuous transitivity.

## NOTES

1. Transitive graphs are discussed in Harary, Norman, and Cartwright (1965: chs. 10 and 11).

2. Davis has thirteen triad types rather than the sixteen given in Figure 3 because he groups types 111U and 111D together, types 021U and 021D together, and types 120U and 120D together.

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