

# Finite-size effects in the dynamics of neutrally buoyant particles in turbulent flow

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The dynamics of neutrally buoyant particles transported by a turbulent flow is investigated for spherical particles with radii of the order of the Kolmogorov dissipative scale or larger. The pseudo-penalization spectral method that has been proposed by Pasquetti *et al.* (*Appl. Numer. Math.*, vol. 58, 2008, pp. 946–954) is adapted to integrate numerically the simultaneous dynamics of the particle and of the fluid. Such a method gives a unique handle on the limit of validity of point-particle approximations, which are generally used in applicative situations. Analytical predictions based on such models are compared to result of very well-resolved direct numerical simulations. Evidence is obtained that Faxén corrections reproduce dominant finite-size effects on velocity and acceleration fluctuations for particle diameters up to four times the Kolmogorov scale. The dynamics of particles with larger diameters is consistent with predictions obtained from dimensional analysis.

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## 1. Introduction

A large number of natural and engineering situations involves the transport of spherical finite-size particles by a fully developed turbulent flow. This includes the formation of planets in the early solar system, rain formation in clouds, the coexistence between several species of plankton and many industrial settings encountered in chemistry and material processes. An important feature of such particles is that they do not follow exactly the fluid motion but have inertia, a property that leads to the development of inhomogeneities in their spatial distribution (see Squires & Eaton 1991; Balkovsky, Falkovich & Fouxon 2001; Bec *et al.* 2007) or to the enhancement of the rate at which they collide (see Falkovich, Fouxon & Stepanov 2002; Wilkinson, Mehlig & Bezuglyy 2006; Zaichik, Simonin & Alipchenkov 2006; Bec *et al.* 2010). The modelling of such particles generally assumes that their diameter  $d_p$  is much smaller than the smallest active length scale of the flow, i.e. the Kolmogorov scale  $\eta$ , so that they can be approximated by points (see Maxey & Riley 1983; Gatignol 1983). Modelling situations where  $d_p \gtrsim \eta$  relies on the use of various empirical laws (as reviewed e.g. in Clift, Grace & Weber 1978). Generally such laws are obtained by considering a particle suspended in a mean laminar flow and interacting only through the turbulent wake that it creates, but not with a fully developed turbulent environment maintained by an external energy input.

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Recent experimental developments have triggered a renewal of interest in the understanding and quantification of finite-size effects in the motion of particles in a turbulent flow (Qureshi *et al.* 2007; Volk *et al.* 2008; Xu & Bodenschatz 2008; Calzavarini *et al.* 2009). These works addressed in particular the problem of delimiting the domain of validity of the point-particle models that are largely used in applicative fields, and to understand which corrective terms give dominant corrections. Such questions remain largely open because of the difficulty in constructing analytically the fluid flow perturbed by the presence of the spherical particle. In the following, we briefly review the equations that govern the coupled dynamics of the flow and the particle. In an incompressible fluid with kinematic viscosity  $\nu$  and subject to an external volumic forcing strain tensor  $\mathbf{F}$ , the velocity field  $\mathbf{u}$  solves the Navier–Stokes equation

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho_f} \nabla p + \nu \nabla^2 \mathbf{u} + \nabla \cdot \mathbf{F} + \mathbf{g}, \quad \nabla \cdot \mathbf{u} = 0, \quad (1.1)$$

where  $\mathbf{g}$  designates the acceleration of gravity. The equation for the fluid is supplemented by a non-slip boundary condition

$$\mathbf{u}(\mathbf{X}_p(t) + (d_p/2)\mathbf{n}, t) = \mathbf{V}_p(t) + (d_p/2)\boldsymbol{\Omega}_p(t) \times \mathbf{n}, \quad \forall \mathbf{n} : |\mathbf{n}| = 1 \quad (1.2)$$

at the surface  $\partial\mathcal{B}$  of the spherical particle.  $\mathbf{X}_p(t)$  denotes here the trajectory of the centre of the particle,  $\mathbf{V}_p(t)$  its translational velocity and  $\boldsymbol{\Omega}_p(t)$  its rotation rate. The motion of the particle is determined by Newton’s second law

$$m_p \frac{d\mathbf{V}_p}{dt} = (m_p - m_f) \mathbf{g} + \int_{\partial\mathcal{B}} \nabla \cdot \mathbb{T} \, d\mathcal{V} = (m_p - m_f) \mathbf{g} + \int_{\partial\mathcal{B}} \mathbb{T} \cdot d\mathcal{S}, \quad (1.3)$$

where  $m_p = (\pi/6)\rho_p d_p^3$  is the particle mass (with  $\rho_p$  its mass density),  $m_f = (\pi/6)\rho_f d_p^3$  the mass of the displaced fluid,  $\mathbb{T} = -p \mathbb{I}_3 + \mu(\nabla \mathbf{u} + \nabla \mathbf{u}^\top) + \rho_f \mathbf{F}$  denotes the fluid stress tensor,  $\mathbb{I}_3$  is the identity and  $\mu = \rho_f \nu$  the dynamic viscosity. In addition, the sphere rotation rate  $\boldsymbol{\Omega}_p$  changes according to the conservation of angular momentum

$$\mathbf{I} \frac{d\boldsymbol{\Omega}_p}{dt} = \frac{m_p}{10} d_p^2 \frac{d\boldsymbol{\Omega}_p}{dt} = \frac{d_p}{2} \int_{\partial\mathcal{B}} \mathbf{n} \times (\mathbb{T} \cdot d\mathcal{S}), \quad (1.4)$$

where  $\mathbf{n}$  denotes the outward pointing unit vector normal to the surface and where we have assumed that the mass moment of inertia tensor  $\mathbf{I}$  is that of a uniform solid sphere. Solving the system (1.1)–(1.4) is a difficult task as it involves a nonlinear partial differential equation for the fluid, which is coupled to a moving boundary condition on the sphere. Analytical treatments of such a complex dynamics has only been done when neglecting nonlinearities in the flow motion at the scale of the particle, so that (1.1) reduces to the Stokes equation (this leads to the usual point-particle models).

This study focuses on neutrally buoyant particles, i.e.  $\rho_p = \rho_f$ . This case is of interest for applications to problems of plankton dynamics in the ocean or of some types of ice crystals in clouds. The goal is here to give a complete description of the dynamical properties of particles with sizes of the order of the Kolmogorov dissipative scale  $\eta$ . The paper is organized as follows. In §2, we consider the model given by the point-particle approximation. We show that in the case of neutrally buoyant particles, first-order finite-size effects are not due to particle inertia but purely stem from Faxén corrections. They intervene in the particle dynamics as  $(d_p/\lambda)^2$ , where  $\lambda$  designates the Taylor microscale. These results are validated numerically in §3 thanks to the use of a new dynamical pseudo-penalization technique that has the advantage of allowing one to use a spectral code in order to integrate the Navier–Stokes equation with

the proper boundary conditions. We show that, both for velocity and acceleration statistics, finite-size effects become noticeable for  $d_p \simeq 3\eta$  and that first-order Faxén corrections are relevant up to  $d_p = 4\eta$ . For  $d_p \gtrsim 4\eta$ , the particle dynamics is dominated by inertial-range physics. We also present results on acceleration time correlation that confirm this fact. Finally, §4 is dedicated to concluding remarks and perspectives.

## 2. Point-particle approximation

The derivation of point-particle models relies on the assumption that the perturbation of the surrounding flow by particles is well described by the Stokes equation (see Gatignol 1983; Maxey & Riley 1983; Auton, Hunt & Prud'homme 1988). This assumption clearly requires, on the one hand, that the fluid velocity field does not display a turbulent behaviour at the scale of the particle, and thus that  $d_p \lesssim \eta$  and, on the other hand, that the particle Reynolds number defined with the velocity difference between the fluid and the particle is very small. The motion of a neutrally buoyant particle is then given by

$$\frac{d\mathbf{V}_p}{dt} = \frac{d\mathbf{U}^{\mathcal{V}}}{dt} - \frac{12\nu}{d_p^2}(\mathbf{V}_p - \mathbf{U}^{\mathcal{S}}) + \frac{6}{d_p} \sqrt{\frac{\nu}{\pi}} \int_{-\infty}^t \left( \frac{d\mathbf{U}^{\mathcal{S}}}{ds} - \frac{d\mathbf{V}_p}{ds} \right) \frac{ds}{\sqrt{t-s}}, \quad (2.1)$$

where  $\mathbf{U}^{\mathcal{S}}$  and  $\mathbf{U}^{\mathcal{V}}$  account for Faxén corrections. They are averages of the fluid velocity over the surface and of the fluid acceleration over the volume of the particle, respectively:

$$\mathbf{U}^{\mathcal{S}}(t) = \frac{1}{\pi d_p^2} \int_{\partial\mathcal{B}} \mathbf{u}(\mathbf{x}, t) d\mathcal{S} \quad \text{and} \quad \mathbf{U}^{\mathcal{V}}(t) = \frac{6}{\pi d_p^3} \int_{\mathcal{B}} \mathbf{u}(\mathbf{x}, t) d\mathcal{V}. \quad (2.2)$$

The various forcing terms in (2.1) are, in order of appearance, the combination of the inertia force exerted by the undisturbed flow and the added mass, the Stokes drag and the Basset–Boussinesq history force. As the above approach supposes that the particle is much smaller than the Kolmogorov scale, we also have for turbulent flows that it is well below the Taylor microscale  $\lambda$ . A Taylor expansion of the fluid velocity in the vicinity of the particle centre leads to

$$\left. \begin{aligned} \mathbf{U}^{\mathcal{S}}(t) &= \mathbf{u}(\mathbf{X}_p, t) + (1/40) d_p^2 \nabla^2 \mathbf{u}(\mathbf{X}_p, t) + O[(d_p/\lambda)^4], \\ \mathbf{U}^{\mathcal{V}}(t) &= \mathbf{u}(\mathbf{X}_p, t) + (1/24) d_p^2 \nabla^2 \mathbf{u}(\mathbf{X}_p, t) + O[(d_p/\lambda)^4]. \end{aligned} \right\} \quad (2.3)$$

As the above model for particle dynamics requires that the particle diameter is much smaller than the Kolmogorov scale  $\eta$ , one can show that the Basset–Boussinesq history force gives a contribution much smaller than the Stokes drag and can thus be neglected to leading order. Hence, finite-size neutrally buoyant particles obey asymptotically the minimal model equation

$$\frac{d\mathbf{V}_p}{dt} = \frac{D\mathbf{u}}{Dt}(\mathbf{X}_p, t) - \frac{12\nu}{d_p^2} [\mathbf{V}_p - \mathbf{u}(\mathbf{X}_p, t)], \quad (2.4)$$

where  $D/Dt = \partial_t + \mathbf{u} \cdot \nabla$  denotes the material derivative along fluid tracer trajectories. Note that in this model, the particle size enters only the coefficient of the drag force. However, as it is now shown, such an effect is actually not sufficient to account for leading-order corrections due to the particle finite size. Indeed, following Babiano *et al.* (2000) and introducing the velocity difference between the particle and the fluid

$\mathbf{W}(t) = \mathbf{V}_p(t) - \mathbf{u}(\mathbf{X}_p(t), t)$ , one can easily check that

$$\frac{d\mathbf{W}}{dt} = -\mathbf{W} \cdot \nabla \mathbf{u}(\mathbf{X}_p(t), t) - \frac{12\nu}{d_p^2} \mathbf{W}. \quad (2.5)$$

This implies that  $\mathbf{W}(t) = \exp(-12\nu t/d_p^2) \mathcal{T} \exp[-\int_0^t \nabla \mathbf{u}(\mathbf{X}_p(s), s) ds] \mathbf{W}(0)$ , where  $\mathcal{T} \exp$  denotes the time-ordered exponential. Hence the amplitude of the velocity difference grows exponentially at large time, i.e.  $|\mathbf{W}(t)| \simeq |\mathbf{W}(0)| \exp[-(12\nu/d_p^2 + \lambda_3)t]$ , where  $\lambda_3$  is the smallest Lyapunov exponent associated to  $\mathcal{T} \exp[-\int_0^t \nabla \mathbf{u}(\mathbf{X}_p(s), s) ds]$ . Because of the fluid flow incompressibility implying a vanishing sum of the Lyapunov exponents,  $\lambda_3 \leq 0$ . However, when  $d_p$  is sufficiently small, i.e. when  $d_p \leq \sqrt{12\nu/|\lambda_3|}$ , the exponential growth rate of  $|\mathbf{W}|$  is negative and the particle velocity relaxes to that of the fluid. Estimating the value of  $\lambda_3$  requires in principle to evaluate the Lyapunov exponents associated to the fluid flow strain along particle trajectories. However, because of the exponential relaxation of the particle velocities to that of the fluid, these exponents are exactly those computed along tracer trajectories. The latter have been evaluated in direct numerical simulations (see e.g. Bec *et al.* 2006) and, once normalized by the inverse of the Kolmogorov eddy turnover time  $\tau_\eta$ , depend weakly upon the Reynolds number of the flow: for  $Re_\lambda$  varying from 65 to 185, one observes  $\tau_\eta \lambda_3 = 0.190\% \pm 2\%$ . To summarize, this shows that the minimal model (2.4) exactly matches tracer dynamics for sizes smaller than a fixed threshold, i.e. for

$$\Phi = d_p/\eta \leq \Phi^* = \sqrt{12/(\tau_\eta \lambda_3)} \approx 8. \quad (2.6)$$

Hence, for  $\Phi \leq \Phi^*$  finite-size effects are not related to inertia but can only stem from terms that were neglected in this model. This observation explains why neutrally buoyant particles whose dynamics are approximated by the point-particle model (2.4) do not display any clustering properties, as observed by Calzavarini *et al.* (2008).

Let us now estimate the contribution from terms that were neglected, namely from the Faxén corrections and the Basset–Boussinesq history term. For this, we follow the approach of Maxey (1987) developed for small Stokes numbers and write the following perturbative Ansatz  $\mathbf{V}_p(t) = \mathbf{u}(\mathbf{X}_p(t), t) + (d_p/\lambda)^\alpha \mathbf{f}(t) + O[(d_p/\lambda)^\alpha]$ , where the order  $\alpha$  and the function  $\mathbf{f}$  are to be determined. Inserting this form in (2.1) and (2.3), one obtains that the first-order terms originate from Faxén corrections to the Stokes drag, so that  $\alpha = 2$  and

$$\mathbf{V}_p(t) = \mathbf{u}(\mathbf{X}_p(t), t) + (d_p^2/40) \nabla^2 \mathbf{u}(\mathbf{X}_p(t), t) + O[(d_p/\lambda)^4]. \quad (2.7)$$

Note that the synthetic velocity field defined above is divergence-free, implying that no effect of particle preferential concentration can be detected with first-order corrections. This asymptotic form (2.7) implies that the particle velocity variance satisfies

$$\langle |\mathbf{V}_p|^2 \rangle - \langle |\mathbf{u}|^2 \rangle \simeq (d_p^2/20) \langle \mathbf{u} \cdot \nabla^2 \mathbf{u} \rangle = -(d_p^2/20)(\varepsilon/\nu) = -(\langle |\mathbf{u}|^2 \rangle/100) (d_p/\lambda)^2, \quad (2.8)$$

where  $\varepsilon$  is the mean turbulent rate of kinetic energy dissipation. As for the variance of particle acceleration, one obtains from the time derivative of (2.7):

$$\left\langle \left| \frac{d\mathbf{V}_p}{dt} \right|^2 \right\rangle - \left\langle \left| \frac{D\mathbf{u}}{Dt} \right|^2 \right\rangle \simeq -\frac{d_p^2}{20} \left\langle \left\| \frac{D\nabla \mathbf{u}}{Dt} \right\|^2 \right\rangle = -\frac{d_p^2}{20} \left\langle \frac{1}{\rho_f^2} (\nabla^2 p)^2 + \nu^2 \|\nabla^2 \nabla \mathbf{u}\|^2 \right\rangle, \quad (2.9)$$

where  $\|\cdot\|$  denotes the tensorial Frobenius norm, i.e.  $\|\mathbf{M}\|^2 = \text{trace}(\mathbf{M}^T \mathbf{M})$ . Note that the leading terms that appear when writing  $d\mathbf{u}/dt$  in terms of  $D\mathbf{u}/Dt$  are

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$Re_\lambda$	$u_{rms}$	$\varepsilon$	$\nu$	$\delta x$	$\delta t$	$\eta$	$\tau_\eta$	$\lambda$	$L$	$T_L$	$N^3$
32	0.17	$4.5 \times 10^{-3}$	$3 \times 10^{-3}$	$1.23 \times 10^{-2}$	$8 \times 10^{-3}$	$5 \times 10^{-2}$	0.8	0.56	1.2	6.5	512 <sup>3</sup>

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TABLE 1. Parameters of the numerical simulations.  $Re_\lambda = \sqrt{15u_{rms}L/\nu}$ : Taylor–Reynolds number,  $u_{rms}$ : root-mean-square (r.m.s.) velocity,  $\varepsilon$ : mean kinetic energy dissipation rate,  $\nu$ : kinematic viscosity,  $\delta x$ : grid spacing,  $\delta t$ : time step,  $\eta = (\nu^3/\varepsilon)^{1/4}$ : Kolmogorov dissipation length scale,  $\tau_\eta = (\nu/\varepsilon)^{1/2}$ : Kolmogorov time scale,  $\lambda = u_{rms}\sqrt{15\nu/\varepsilon}$ : Taylor microscale,  $L = u_{rms}^3/\varepsilon$ : integral scale,  $T_L = L/u_{rms}$ : large-eddy turnover time,  $N^3$ : number of collocation points.

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exactly equal to the quadratic (vortex stretching) terms appearing in the passage from  $\nabla \mathbf{D}\mathbf{u}/Dt$  to  $D\nabla \mathbf{u}/Dt$ . To summarize we expect at small diameters that finite-size effects due to Faxén corrections materialize as a falloff of the two above-mentioned dynamical properties of particles that behave quadratically as a function of the particle diameter with a coefficient given by Eulerian averaged quantities.

### 3. Numerical results using a pseudo-penalization method

In order to assess numerically the effect of the particles' finite size on their dynamics, several methods have been used. These range from the use of the Lattice–Boltzmann approach (Ten Cate *et al.* 2004), of coupling the outer flow to known solutions to the Stokes equation near the boundary (Zhang & Prosperetti 2005) to the used of finite-difference schemes on coupled grids (Burton & Eaton 2005). We decided to use a pseudo-spectral code that has been adapted to non-trivial geometries by using a pseudo-penalization strategy. The purely spectral part of the parallel code LaTu, which was already used to investigate Lagrangian turbulence by Homann *et al.* (2007), solves accurately the incompressible Navier–Stokes equations. This method treats the evolution of the fluid velocity field in Fourier space and computes convolutions arising from the nonlinear terms in physical space. The Fourier transformations are performed by the P3DFFT library (Parallel Three-Dimensional Fast Fourier Transforms, <http://www.sdsc.edu/us/resources/p3dfft>). The domain is a triple-periodic cube. This method allows for high accuracy, precise control of the physical parameters and numerical efficiency. In order to maintain a statistically steady state we force the flow by prescribing the energy content of the Fourier vectors with moduli 1 and 2. The energy content of each of these two shells is kept constant while the individual amplitudes and phases are evolved piecewise linearly in time between several random configurations separated by a time  $10 T_L$ . The advantages of such a forcing are two-fold: it allows one to achieve a statistically isotropic large-scale flow and limits the fluctuations of the total kinetic energy to only approximately 10% of its mean value. The turbulent characteristics of the flow generated in this way are summarized in table 1.

In order to impose the no-slip boundary condition on the surface of the spherical particle, we follow Pasquetti, Bwemba & Cousin (2008) and use a pseudo-penalization method, which consists in imposing a strong drag to the fluid velocity at the particle location, so that it relaxes quickly to the particle solid motion. The hydrodynamical forces acting on the particle are computed by a Riemann approximation of the integrals appearing in (1.3) and (1.4) on a homogeneous grid of discrete points located on the surface of the sphere. The value of pressure at these points is computed by tri-cubic interpolation. The surface integral of the fluid velocity gradient is computed from evaluating the average velocity on spherical shells surrounding the particle.

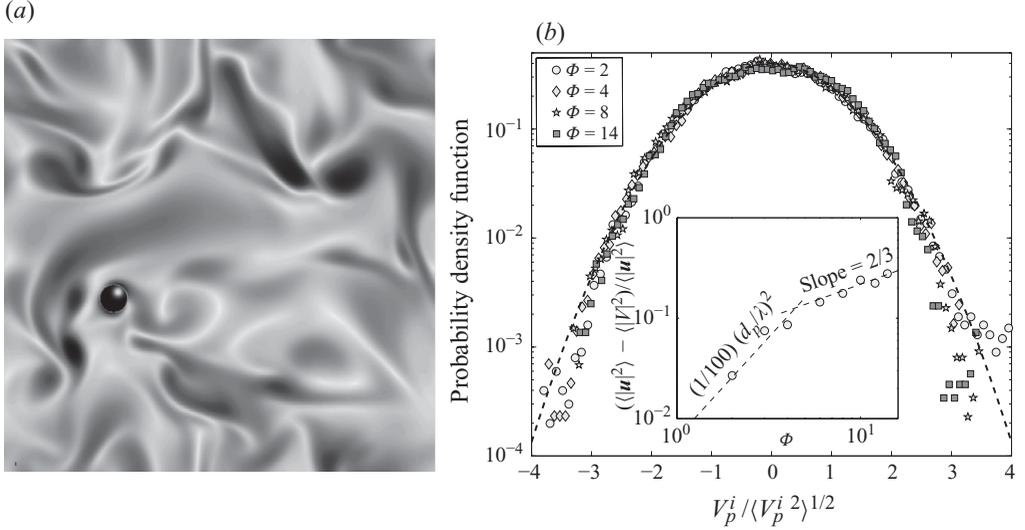


FIGURE 1. (a) Modulus of the vorticity in a slice of the domain that is passing through the centre of the embedded particle (dark = high vorticity, light = low vorticity); the particle diameter is here  $d_p = 8\eta$ . (b) Normalized probability density function of the particle velocity for various particles sizes, as labelled; the bold dash line corresponds to a Gaussian distribution. Inset: deviation of the particle velocity variance  $\langle |V_p|^2 \rangle$  from the fluid value, as a function of the non-dimensionalized particle diameter  $\Phi = d_p / \eta$ ; the two dashed line represent the deviation (2.8) from the fluid root-mean-square (r.m.s.) velocity that is expected to stem from Faxén corrections and a behaviour  $\propto d_p^{2/3}$ , respectively.

At the moment, the simulations are limited to a single particle in order to prevent the individual dynamical properties from being contaminated by particle–particle hydrodynamical interactions. Notice however that the code is very well adapted to situations involving several particles. Because only a single isolated particle is considered in the flow, the statistical convergence of particle-related quantities requires to perform averages over very long times. Each simulation for a single value of the particle diameter required to integrate the flow over more than 300 large-eddy turnover times. Eight different particle diameters  $d_p$  are considered within the range  $2\eta$ – $14\eta$ . As the pseudo-penalization technique requires several grid points inside the object in order to correctly impose the boundary conditions, the Kolmogorov dissipative scale  $\eta$  is equal to four times the grid spacing, so that the smallest particle is resolved with nine grid points along its diameter. Resolving well-dissipative scales requires the use of double floating point precision. Because of the large spatial resolution and the long time integration which are required, the simulations are very computationally demanding: this work took approximately four million of single processor CPU hours. Figure 1(a) shows the typical vorticity field in a cut-plane passing through the centre of the particle. Note that the signature of a turbulent wake is visible on the right-hand side of the particle.

To benchmark these simulations, a run with the same parameters as those shown in table 1 but without any finite-size particle has been performed. In this simulation, we have integrated the motion of passive point particles with a dynamics obeying (2.4) and of passive tracers. It is worthwhile mentioning here that turbulent fluid statistical quantities are observed not to depend on the presence of a particle, up to the statistical

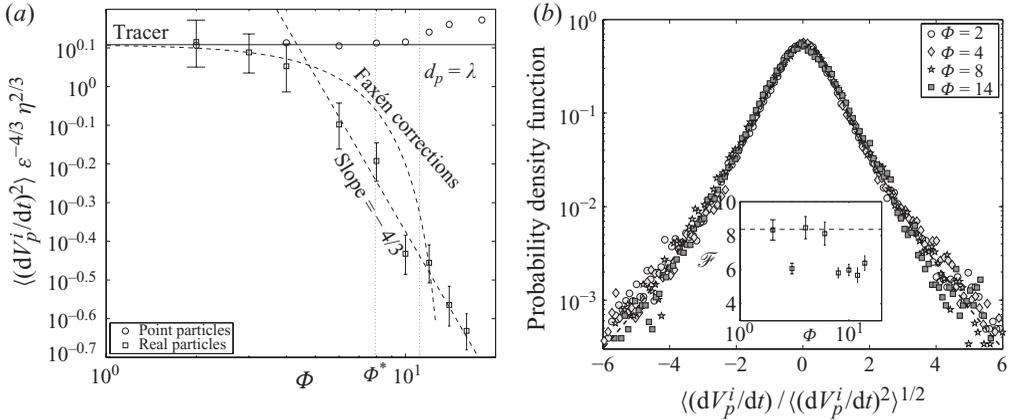


FIGURE 2. (a) Acceleration variance of real spheres and of point particles as a function of the non-dimensionalized particle diameter  $\Phi = d_p/\eta$ ; error bars correspond to an estimation of the standard deviations. The two dotted vertical line indicate the Taylor microscale  $\lambda$  and the critical value (2.6) above which point particles deviate from fluid tracers. The dashed curve corresponds to the behaviour (2.9) predicted from Faxén corrections. (b) Normalized probability density function of the componentwise particle acceleration for various particle diameter, as labelled. The bold dashed line on which data almost collapse corresponds to the log-normal fit of experimental data proposed by Mordant, Crawford & Bodenschatz (2004) with the value of flatness  $\approx 8.37$  as observed in Qureshi *et al.* (2007). Inset: variation of the acceleration flatness  $\mathcal{F}$  as a function of the particle size. The dashed horizontal line correspond to the value obtained from tracers.

fluctuations due to finite time averages. The particle-free simulation serves thus as a reference to estimate tracer statistics and Eulerian averages.

Figure 1(b) shows the probability density function (PDF) of the particle velocity components for various particle diameters. Once normalized by their standard deviations, these PDFs almost collapse on top of each other and deviate very weakly from a Gaussian distribution. The measured variance of the particle velocity that is represented in the inset, decreases as a function of the particle size. For small diameters, i.e. for  $\Phi = d_p/\eta \lesssim 4$ , the behaviour of the particle velocity variance is very well described by the prediction (2.8) obtained from Faxén corrections. For  $\Phi \gtrsim 4$ , the deviation from the fluid velocity variance is compatible with the power-law behaviour  $\propto \Phi^{2/3}$  obtained from Kolmogorov 1941 dimensional arguments. This statement has to be taken cautiously as such a behaviour can only be observed over less than the third of a decade in particle diameter. Moreover, in the low-Reynolds-number flow that we are considering here, there is no inertial range in the sense usually defined through velocity spectrum or increments scaling properties. However, the spherical averages of fluid quantities that are involved in particle dynamics seems to be well amenable for dimensional analysis.

We next turn to particle acceleration statistics. Figure 2(a) represents the componentwise normalized variance of the particle acceleration  $a_0 = \langle (dV_p^i/dt)^2 \rangle \epsilon^{-4/3} \eta^{2/3}$  as a function of the non-dimensionalized diameter  $\Phi = d_p/\eta$ , both for the real particles as well as for the minimal point model (2.4). For real spherical particles, one distinguishes, as in the case of velocity variance, between two behaviours. When  $\Phi = d_p/\eta \lesssim 4$ , finite-size effects in the acceleration variance are very well captured by Faxén corrections and are very close to the prediction (2.9). Note that, thanks to their isotropic form, the subleading terms appearing in (2.9) could be evaluated here

through the pressure and velocity spectra. When  $\Phi \gtrsim 4$ , one obtains a behaviour that is compatible with  $a_0 \propto d_p^{-4/3}$ . As argued by Qureshi *et al.* (2007), the variance of finite-size particle acceleration is related to that of the fluid pressure integrated over a sphere of diameter  $d_p$ . The power  $-4/3$  that is observed here differs from the value  $-2/3$ , which was measured by Qureshi *et al.* (2007). However, as already stressed for instance in Bec *et al.* (2007), pressure scaling in low Reynolds number flows is often dominated by sweeping, leading to a behaviour of the pressure increments  $|p(x + \ell) - p(x)| \sim \ell^{1/3}$ . While no scaling of pressure can be detected in the present simulations and only a limited range of particle diameter could be investigated, one cannot exclude that acceleration variance obeys a  $-4/3$  law which is compatible with a sweeping-dominated pressure spectrum. Another observation is that numerics confirm the presence of the threshold (2.6) predicted in the previous section for the minimal point-particle model: indeed the numerical integration of point particles obeying (2.4) shows that when  $\Phi < 8$ , the acceleration variance of the latter is undistinguishable from that of tracers. For  $\Phi > 8$ , the point-particle model gives an enhancement of acceleration, which is incompatible with measurements done with real particles at the Reynolds number considered here. This stresses the irrelevance of such a model in the case of neutrally buoyant particles. Note finally that in the case of tracers, the constant  $a_0$  is known to show a Reynolds number dependence  $a_0 \propto R_\lambda^{1/2}$  (see e.g. Voth *et al.* 2002). The measured value of approximately 1.3 is in good agreement with the value that was measured experimentally by Qureshi *et al.* (2007).

Figure 2(b) represents the PDF of acceleration components normalized to unity variances for various values of the particle diameter. As already stressed in Qureshi *et al.* (2007), the dependence upon  $\Phi = d_p/\eta$  is very weak. All distributions seem to collapse up to the statistical error seen in the tails. The inset of the figure shows the acceleration flatness  $\mathcal{F} = \langle (dV_p^i/dt)^4 \rangle / \langle (dV_p^i/dt)^2 \rangle^2$  as a function of the particle diameter. The observed decrease from  $\mathcal{F} \approx 8.5$  to  $\mathcal{F} \approx 6$  at  $d_p \approx 6\eta$  should be taken cautiously as it might correspond to a lack of statistics.

Other measurements relate to two-time statistical properties of particles. Figure 3 represents the acceleration time correlation

$$C(\tau) \equiv \langle (dV_p^i/dt)(t + \tau) (dV_p^i/dt)(t) \rangle / \langle (dV_p^i/dt)^2 \rangle \quad (3.1)$$

as a function of the time lag  $\tau$  for various values of the particle diameter. The numerical measurements reported here are in good agreement with the results of Calzavarini *et al.* (2009). Surprisingly, one observes that  $C(\tau)$  deviates only very weakly from the tracer acceleration temporal correlation for diameters less than  $4\eta$ , that is when Faxén corrections are expected to be of relevance to capture first-order finite-size effects. This behaviour is even clearer when looking at the diameter dependence of the correlation time for particle acceleration. For this we follow Calzavarini *et al.* (2009) and introduce the integral time

$$T_I \equiv \int_0^{T_0} C(\tau) d\tau, \quad (3.2)$$

where  $T_0$  is the first zero-crossing time. The inset of figure 3 represents  $T_I/\tau_\eta$  as a function of  $\Phi = d_p/\eta$ . When  $\Phi \lesssim 4$ , this integral correlation time is, up to numerical errors, undistinguishable from the value obtained for tracers. When  $\Phi \gtrsim 4$ , the correlation time increases much faster as a function of  $\Phi$  in a manner that is compatible with the power-law behaviour  $T_I \sim \Phi^{2/3}$ . This relates to dimensional

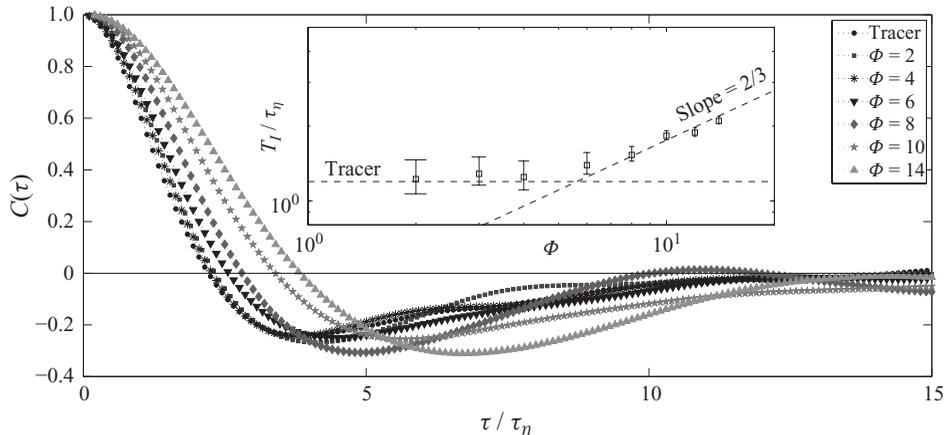


FIGURE 3. Componentwise acceleration time correlation  $C(\tau)$  defined in (3.1) for various particle sizes, as labelled. Inset: integral correlation time  $T_I$  defined from (3.2) as a function of the non-dimensionalized particle diameter  $\Phi = d_p/\eta$ .

analysis as the relevant time scale at the particle size relates to the eddy turnover time  $\sim \varepsilon^{-1/3} d_p^{2/3}$ .

#### 4. Concluding remarks

In this paper it is shown that first-order finite-size corrections to the dynamics of neutrally buoyant particles are due to Faxén terms and not to particle inertia. This leads to several predictions on turbulent velocity and acceleration second-order statistics. Using a pseudo-penalization spectral method, these predictions have been confirmed numerically for particles with diameters  $d_p$  up to  $4\eta$ . Higher-order statistics of velocity and acceleration seem much less sensitive to the finiteness of the particle sizes, since numerical observation suggest that, once normalized to unit variance, their PDFs collapse on top of each other for various values of the particle diameter.

The irrelevance of particle inertia with respect to Faxén terms at small particle diameters has noticeable consequences. Firstly, it implies that the particle dynamics is very well approximated by the advection by a synthetic flow, which is incompressible to both the leading and the first orders. This means that the effect of preferential concentration onto the dynamics of neutrally buoyant particles is very weak. Secondly, such an observation clearly questions the relevance of inertial-particle models for density contrasts between the particle and the fluid that are close to one. A third consequence is related to the fact that corrective terms apply when the particle diameter is much smaller than the Taylor microscale  $\lambda$  rather than the Kolmogorov dissipative scale  $\eta$ . This fact might partly explain the difficulties in matching experiments and numerical model including Faxén corrections (see, e.g. Calzavarini *et al.* 2009).

For particle diameters larger than  $\approx 4\eta$ , inertial-range physics seems to pull strings. Dimensional estimates given by Kolmogorov 1941 theory seems of relevance. Of course, issues on numerical resolution compelled the current study to a very limited range of particle diameters and to a value of the Reynolds number that is so low that no scaling range can be observed for Eulerian velocity statistics. Studying the dynamics of particles with inertial-range sizes requires working in higher Reynolds number flows. Applying the pseudo-penalization method to this scope is the subject

of on-going work that is mainly focusing on describing the flow modification induced by the presence of the spherical particle.

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