Suspension and Fall of Heavy Particles in Random Two-Dimensional Flow

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We investigate the settling of heavy particles in a steady, two-dimensional random velocity field, and find instances in which particle suspension occurs. This leads to a bimodal velocity distribution that may explain some apparently conflicting results reported in the literature. The bimodal distribution is typically smeared out by a time dependence of the ambient flow but, if the time variation is slow, the settling rates of some particles will be as well. The resulting broadbanded velocity distribution of the settling particles will have significance for processes such as rain drop formation, in which the spread of particle velocities affects the statistics of particle collisions.

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Transport of small heavy particles in turbulent flows is significant in problems involving the motions of aerosols and cloud droplets in the atmosphere, sedimentation in rivers, and plankton migration in the ocean. An important role for the collisions and coalescence of sedimenting dust particles arises in studies of the formation of planetesimals [1]. Theoretical understanding of the dynamics of non-neutrally buoyant, finite-sized impurities has accordingly been widely sought, beginning with Stommel's [2] description of how heavy particles can be permanently suspended in a steady two-dimensional array of eddies. However, when the ambient fluid flow is made time periodic, the particles fall (in a sense chaotically) [3]. More significantly, when the particle inertia is not neglected (as it was in Stommel's work) particles cannot be permanently suspended in a steady cellular flow field [4].

Complicating the ambient flow further enriches the problem. Maxey [5] has found that the average settling velocity of a small rigid spherical particle in homogeneous turbulence increases with particle inertia. His results, which have been parametrized [6], are supported by direct numerical simulation [7,8], and by laboratory experiments [9]. However, some studies indicate that other behavior is possible. Fung [10] observed a net decrease of the average settling velocity in a simulation of particle motion in a Gaussian random velocity field. The investigation by Dávila and Hunt [11] shows that, near steady vortices with horizontal rotation axes, the average settling velocities of particles can be either larger or smaller than the terminal velocity in still fluid, depending on the particle properties. Similar results were obtained in a laboratory experiment on particles sedimenting in grid-generated homogeneous turbulence [12].

Here, we investigate the settling of heavy particles in a turbulent fluid. We confine ourselves to a two-dimensional vertical slice of an idealized three-dimensional turbulent field, neglecting feedback of the particles on the flow. We find that particles can be suspended for long times despite the effects of gravity and inertia when the flow changes slowly. Indeed, the settling speed of individual particles can be very different from the average speed. The latter can be either larger or smaller than the settling velocity in still fluid, depending upon the relative abundance of the particles that stay aloft for long times compared to the particles that are swept into the downdrafts between turbulent eddies. In addition, the velocity distribution of the settling particles can be significantly broadened and even bimodal.

The forces acting on the motion of low concentrations of finite-sized, non-neutrally buoyant particles in a prescribed fluid flow have been surveyed by [13–15]. (Further mathematical background is given in [15–22].) For particles heavier than the displaced fluid, the dynamics reduces to a balance among particle acceleration, Stokes drag, and weight [23]:

$$\frac{d\mathbf{V}(t)}{dt} = \frac{1}{\tau_a}[\mathbf{u}(\mathbf{Y}(t), t) - \mathbf{V}(t) + \mathbf{W}], \tag{1}$$

where $\mathbf{u} = (u, v)$ is the fluid velocity, τ_a is the particle inertial response time, $\mathbf{W} = (0, g\tau_a)$ is the terminal velocity in still fluid, $\mathbf{V} = (U, V)$ is the velocity of the particle, and $\mathbf{Y} = (X, Y)$ is the particle position, with

$$\frac{d\mathbf{Y}(t)}{dt} = \mathbf{V}(t). \tag{2}$$

Equations (1) and (2) define a dissipative system in the four-dimensional phase-space (X, Y, U, V). For heavy particles and a specified fluid velocity field, the divergence of the phase-space velocity is $-1/\tau_a$. We consider heavy particles that are uniformly released in a fluid with a homogeneous, isotropic, and statistically stationary random velocity field $\mathbf{u}(x, y, t)$. This field is prescribed in a vertical, square domain of size L_0, with periodic boundary conditions in both the vertical and the horizontal directions that extend it throughout a vertical plane. The kinetic energy spectrum of the field is chosen as $E(k) = \epsilon^{2/3} k^{-5/3}$ for $2\pi/L_0 < k < k_{\max}$. This choice retains the spectral properties of the inertial range of three-dimensional turbulence, but it ignores the motion along the direction orthogonal to the slice.

When the particle response time τ_a is much smaller than the time scale of the flow evolution, the ambient velocity field can be taken as steady. Otherwise, we use a stochastic differential equation for the Fourier phases to describe the temporal evolution of the velocity field; details of the model are given in the *Appendix*. In the construction of the velocity field, values of the energy dissipation rate ϵ between 300 and 3×10^5 cm^2/s^3 were adopted. If we take a value $\nu = 0.14$ cm^2/s for the kinematic viscosity of the ambient fluid (as for air), we obtain the Kolmogorov length scale, $\lambda_k = \nu^{3/4} \epsilon^{-1/4}$, in the range 0.1–0.6 mm. The Kolmogorov time scale and velocity are then $\tau_k = \nu^{1/2} \epsilon^{-1/2}$ and $v_k = \nu^{1/4} \epsilon^{1/4}$ [25]; $L_0 = 0.1$ m.

In a statistically steady flow, the time-averaged vertical velocity of a particle becomes stationary after a transient whose duration depends on the initial conditions and the particle's inertial response time. We discard the initial transients and compute the properties of particle motion at times larger than $t_0 = 100\tau_a$ beyond particle release. Of interest are the Lagrangian settling rates, defined as the average vertical velocity over the particle trajectories, $V_L = L_0 \langle \tau^{-1} \rangle$, where τ is the time a particle takes to cover the vertical distance L_0, and the bulk settling rate, defined as the inverse of the average time that particles take to cover a unit vertical distance, $V_B = L_0/\langle \tau \rangle$. These two quantities are in general different [11]. We characterize the effect of the ambient flow on the particle settling rate by $L_0/\tau - W$; a positive value indicates that the particle falls faster than in still fluid. Figure 1 shows the distribution $p(L_0/\tau - W)$ in a random steady flow with $W/v_k = 0.75$, for different values of the ratio τ_a/τ_k. The mean value of this distribution can be either positive or negative, indicating that, on average, the settling rate of heavy particles in a turbulent flow can be either larger or smaller than in still fluid.

We now choose a specific set of $N = 16\,384$ identical particles with $\tau_a/\tau_k = 0.35$, and consider the distribution of the vertical displacements, $\Delta Y_i(t_0, \tau) = [Y_i(t_0 + \tau) - Y_i(t_0)]$, at three different times, namely, $\tau = 10L_0/U_0$, $15L_0/U_0$, and $20L_0/U_0$, where U_0 is the rms turbulent velocity. We obtain a particle distribution that indicates the presence of two distinct populations of particles, as shown in Fig. 2. The value of $(V_L - W)$ calculated on the population of particles with large vertical displacements is $(0.70 \pm 0.02)U_0$, indicating that the average vertical velocity is larger than the terminal velocity in still fluid. Conversely, the value of $(V_L - W)$ for the second population is $(-0.17 \pm 0.06)U_0$. Figure 3 shows that these latter particles move on closed trajectories and have zero time-averaged vertical velocity: These particles are permanently suspended in the flow.

Because of inertia, particles generally do not move along streamlines. When a particle comes to a bend in a streamline, its path curves somewhat less than that of a fluid element would. In a simple circular eddy, the curvature of a closed streamline is the same all the way around

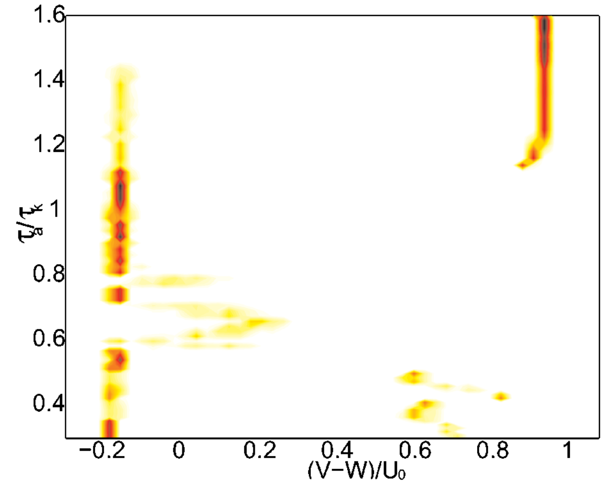

FIG. 1 (color online). Distribution function of $(V - W) = (L_0/\tau - W)$, for different values of τ_a/τ_k and constant $W/v_k = 0.75$. White indicates low or zero density, dark indicates large density.

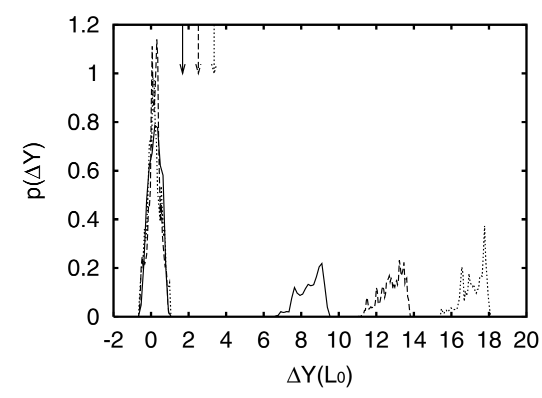

FIG. 2. Probability density function of the vertical displacement $\Delta Y_i(t_0, \tau) = Y_i(t_0 + \tau) - Y_i(t_0)$, for different times: $\tau = 10L_0/U_0$ (solid line), $\tau = 15L_0/U_0$ (dashes), $\tau = 20L_0/U_0$ (dots). Arrows indicate the position that the particles would have if they moved at the Stokes settling velocity W. Here, $\tau_a/\tau_k = 0.35$ and $W/v_k = 0.75$.

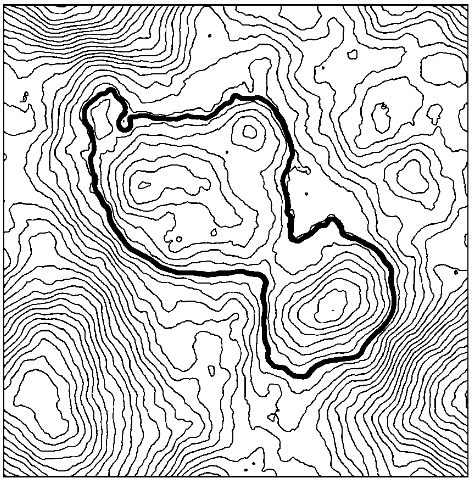

FIG. 3. A closed particle trajectory, superposed on the streamlines of the ambient flow. Here $\tau_a/\tau_k = 0.35$.

[4] and the inertial force flings particles outward. Heavy particles therefore tend to spiral out of circular vortices as shown in Fig. 4(a), so that they may eventually be swept into a downdraft between turbulent eddies and fall. When a vortical structure has a complex shape so that the curvature of its streamlines varies from place to place, an inertial particle may be swept inward or outward in the vortex, depending on the local sign of the streamline's curvature. If the inward and outward particle excursions balance, a closed trajectory can result and the particle may be trapped in the eddy, as in Fig. 4(b).

The balance between inward and outward displacements is not achieved when the time, τ_r, needed for the particle to complete a circuit around the eddy, is less than the inertial response time. In a given turbulent field, smaller eddies give rise to smaller τ_r, and a particle with inertial time τ_a can be trapped only by large enough eddies as measured by the ratio τ_a/τ_r [11]. For $\tau_a/\tau_r \gg 1$, a particle cannot remain suspended. Surprisingly, in given parameter ranges, such closed paths are relatively easy to achieve, perhaps because periodic orbits are dense in chaotic flows.

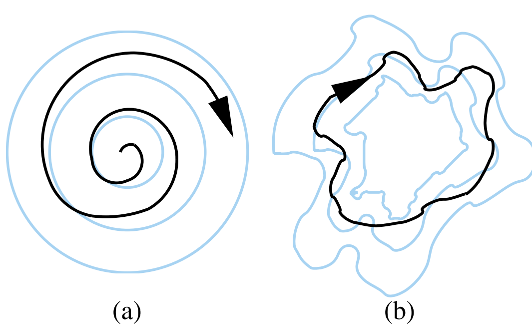

FIG. 4 (color online). Sketch of the inertial bias. Grey thin lines show the streamlines, and solid lines represent inertial particle trajectories. No closed particle trajectory is possible in case (a). In case (b), where the curvature of the streamlines changes sign along the loop, particle suspension is possible.

In Fig. 5, the fraction of suspended particles is shown as a function of the rms turbulent velocity and of the particle inertial time τ_a. For decreasing values of U_0, the ratio between the Stokes terminal velocity W and the flow mean velocity increases, thus inhibiting the suspension of particles. Larger values of τ_a and U_0 correspond to larger values of the ratio τ_a/τ_r, with the same effect. For very intense flows, upward motion of particles for long times has also occurred in our simulations.

Strictly speaking, the foregoing description is invalid in time-dependent flows where, with very few exceptions, permanent suspension cannot occur. However, if the temporal evolution of the flow is slow, some particles have very small settling rates because of the effects of individual eddies [11]. In Fig. 6, we report the distributions of the vertical displacements for the same particles as in Fig. 2, but in a flow evolving with a characteristic time of $180\tau_a$. The bimodal distribution of Fig. 2 is replaced by a unimodal distribution with a large number of particles settling slowly; the fastest particles fall at about the same rate as the nonsuspended particles in the steady flow. For comparison, we note that the width of the distribution of vertical displacements for particles falling in an ambient flow with the same rms velocity as used here but with no spatial correlation (i.e., with a white-noise spectrum) is $0.23L_0$, $0.29L_0$, and $0.33L_0$, respectively, for $\tau = 10L_0/U_0$, $15L_0/U_0$, and $20L_0/U_0$. The distribution of the vertical displacements is thus significantly broadened by the spatial and temporal correlations of the flow.

Our study of the motion of small, heavy, spherical particles in a random two-dimensional flow shows that permanent particle suspension is possible when small-scale turbulence suitably alters the curvature of the streamlines around an eddy. This happens when the inertial bias induces inward and outward particle displacements around the vortical structure that can balance on a closed trajectory, which corresponds to a limit cycle in the four-dimensional particle phase space (X, Y, U, V).

Because of inertial effects, identical particles can have significantly different behavior when moving in a given steady flow. Some particles are swept into the downdrafts between the eddies and settle at rates that are larger than in still fluid. Other particles remain suspended, moving on closed trajectories, so that their vertical motions are oscillatory. These two different types of behavior result

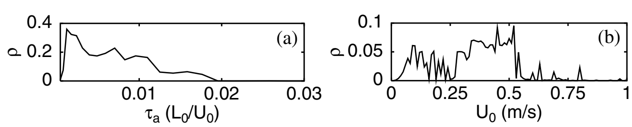

FIG. 5. Fraction ρ of suspended particles as a function of (a) particle inertial time ($U_0 = 0.4$ m/s), and (b) rms velocity of the fluid field ($\tau_a = 0.011L_0/U_0$).

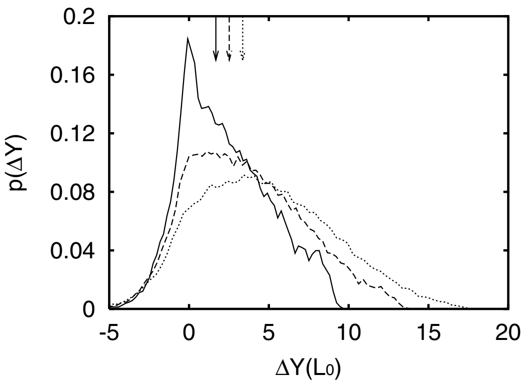

FIG. 6. Same as Fig. 2, but for a time-evolving flow.

in a bimodal distribution of vertical displacements. As a result, the mean settling velocity can be either larger or smaller than the terminal velocity in still fluid depending on the fraction of suspended particles. The bulk settling velocity is affected by the existence of suspended particles and it is not a useful indicator of the sedimentation rate of the falling particles.

When the flow is unsteady, permanent suspension of particles becomes very unlikely. However, if the time scale for the evolution of the vortical structures is larger than the characteristic time for the settling process, the distribution of the vertical displacements at a given time becomes much broader than in the case of particles moving in a spatially uncorrelated flow. The wide range of settling speeds can induce an increase in the particle collision rate and so render coalescence processes more efficient (see also [26]). This can play a role in rain drop formation and the growth of planetesimals. The reduced falling velocity of some particles in a turbulent flow can also be important in processes such as the sinking of phytoplankton in the upper layer of the ocean; a small settling velocity allows for a long time interval spent by heavy living organisms in the surface euphotic region, where they can reproduce.

Appendix.—To obtain a time-evolving, statistically stationary turbulent flow, we keep the energy spectrum constant and vary only the Fourier phases, ϕ_n. The evolution equation for the phase ϕ_n is the stochastic differential equation,

$$d\phi_n = a(\phi_n)dt + b(\phi_n)d\xi(t),$$

where $d\xi(t)$ is a Wiener process, such that $\langle d\xi(t)\rangle = 0$, $\langle d\xi(t)d\xi(t')\rangle = \delta(t - t')dt$, and a and b are generic functions of the phase ϕ_n. The choice

$$a(\phi_n) = -\frac{\phi_n}{\tau_n}; \qquad b(\phi_n) = \sqrt{\frac{\pi^2}{4\tau_n} - \frac{\phi_n^2}{\tau_n}}$$

ensures that the distribution function $p(\phi_n, t)$ is a constant in the domain of definition of the phase ϕ_n, $[-\pi/2{:}\pi/2]$, and is independent of time. The correlation time scale of the phase ϕ_n is defined as $\tau_n = \epsilon^{-1/3}k_n^{-2/3}$, as in Kolmogorov theory.

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