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On universality of geometrical invariants in turbulence—Experimental results

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Experimental results on probability distribution functions (pdf's) of full dissipation ϵ, enstrophy ω^2, and enstrophy generation $\omega_i\omega_j s_{ij}$ in two different turbulent flows: turbulent grid flow ($\mathrm{Re}_\lambda=74$) and turbulent jet center ($\mathrm{Re}_\lambda=880$) demonstrate the possibility of universal behavior of the pdf's of these quantities.

Geometrical invariants, such as full dissipation $\epsilon=s_{ij}s_{ij}$ (s_{ij} is the rate of strain tensor), enstrophy ω^2, enstrophy generation $\omega_i\omega_j s_{ij}$, etc., are the most appropriate for studying physical processes in turbulent flows[1,2,10] and are of special interest in the context of universal properties of turbulence. These quantities—in contradistinction of individual velocity derivatives and their noninvariant combinations—remain invariant under arbitrary transformations of the system of reference and change only under transformations of time. The invariance property of these quantities allows one to hope that in isotropic (locally isotropic) turbulence the probability distribution functions (pdf's) of these invariants will be insensitive to both the particular type of flow and to the value of Reynolds number at least in some range. There is an increasing interest in pdf's of velocity derivatives in turbulent flows (see, for example, Refs. 1–5 and references therein). Some theoretical models have been based on quantities invariant of the system of reference while others worked with noninvariant combinations of velocity derivatives or just individual derivatives. This problem had no experimental basis since up to recently there have been performed no measurements of all the nine velocity gradients.

We report in this note results on pdf's of geometrical invariants such as enstrophy, full dissipation and enstrophy generation in a grid flow at $\mathrm{Re}_\lambda=74$ (Ref. 1) and in the center of a circular jet at $\mathrm{Re}_\lambda=880$ (Ref. 14). The experimental techniques using multihot-wire probes are described in Ref. 1 and have been used in both experiments with the only difference that a 12 hot wire probe (3 arrays $\times$4 wires) have been used, while in the jet flow a 21 wire (5 arrays$\times$4 wires and a cold wire) have been used.

The pdf's of enstrophy ω^2 and full dissipation for both flows are shown in Figs. 1(a) and 1(b) separately, since they are indistinguishable when plotted on one figure. Note that the flows are different and the Taylor microscale Reynolds number in the jet flow is more than an order of magnitude larger than that in the grid flow. It has been shown in Ref. 5 that at moderate Reynolds numbers the pdf of ϵ at large ϵ has the form

$$P(\epsilon^{1/2})\sim\exp(-\alpha\epsilon^{1/2}/\langle\epsilon\rangle^{1/2}). \tag{1}$$

Figures 1(a) and 1(b) have been plotted in lin–log coordinates, in which the relation (1) becomes a straight line. These straight lines are shown Figs. 1(a) and 1(b). Similarly we checked the relation

$$P(|\omega|)\sim\exp(-\beta|\omega|/\langle\omega^2\rangle^{1/2}). \tag{2}$$

Since the two flows are essentially different in geometry and Reynolds numbers one can expect that experimentally obtained $\alpha=3.32$ and $\beta=2.56$ in both flows are close to some universal values. This claim is supported by the results of numerical simulations described in Ref. 6—the values of α and β we obtained using their data are precisely the same. The inequality $\alpha\neq\beta$ seems to be an interesting theoretical problem. It is noteworthy in this context that close to the "edge" of the jet the values of α and β are almost the same and very close the value of β in the grid flow and the jet center (see Fig. 2).

Perhaps one of the most interesting quantities in turbulent flows is the enstrophy generating term $\sigma=\omega_i\omega_j s_{ij}$. pdf's of this invariant are shown in Figs. 3(a), and 3(b). Again, the lin-log coordinates have been chosen in order to reveal the relation

$$P(\sigma^{1/3})\sim\exp[-\gamma(\sigma/\langle\omega^2\rangle\langle\epsilon\rangle^{1/2})^{1/3}]. \tag{3}$$

The straight lines in Figs. 3(a) and 3(b) correspond to the relation (3) with $\gamma=\mathrm{const}$. As can be expected, the value of $\gamma=\gamma_1$, for $\sigma>0$ is different from $\gamma=\gamma_2$ for $\sigma<0$ and $\gamma_2>\gamma_1$, so that $\langle\sigma\rangle$ is an essentially positive quantity. We

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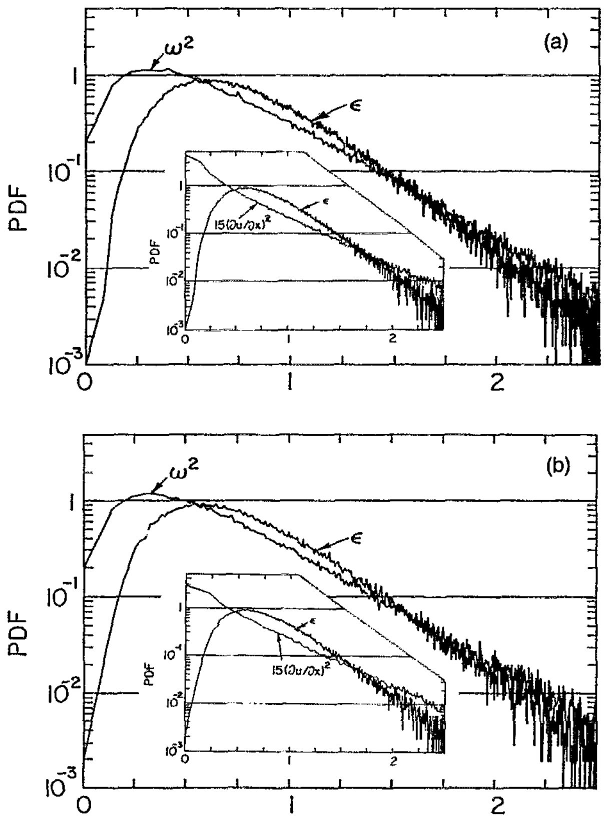

FIG. 1. Probability distribution functions of the square root of normalized enstrophy ω^2 and full dissipation ϵ for (a) turbulent grid flow, $Re_\lambda=74$ (Ref. 1) and (b) turbulent jet center, $Re_\lambda=880$ (Ref. 14). At the inserts of both figures the squared individual derivative $15\ (\partial u/\partial x)^2$ is shown in comparison to the pdf of ϵ to demonstrate the qualitative difference in their behavior.

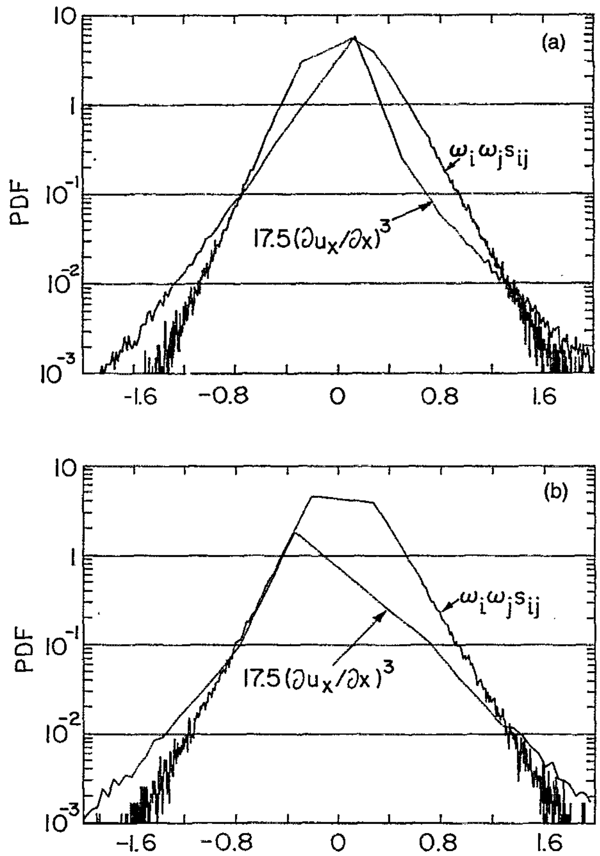

FIG. 3. Probability distribution functions of the cubic root of the normalized enstrophy generating term $\sigma=\omega_i\omega_j s_{ij}$ for (a) turbulent grid flow, $Re_\lambda=74$ (Ref. 1) and (b) turbulent jet center $Re_\lambda=880$ (Ref. 14). In both figures is also shown the pdf of the noninvariant quantity $17.5\ (\partial u/\partial x)^3$.

tend to relate this asymmetry with the true irreversibility of turbulent flows, since the invariant σ changes its sign under the transformation $t\rightarrow -t$.

The phenomenon of positiveness of $\langle\sigma\rangle$ was first discovered by Taylor in 1938[7] (see also Ref. 8). It can be seen as one of the manifestations of the asymmetry of pdf of σ. A closely related phenomenon of strict alignment between ω_i and $\omega_j s_{ij}$ has been discovered experimentally in Refs. 1 and 14 for the flows discussed here and numerically in Ref. 9. All these effects are the consequence of the prevalence of vortex stretching over compressing. It is not clear yet[10] how these phenomena are related to the recently discovered[11] alignment between vorticity and the intermediate eigenvector of the rate of strain tensor, which have been also observed experimentally in Refs. 1 and 14 (see also subsequent numerical experiments in Refs. 12 and 13).

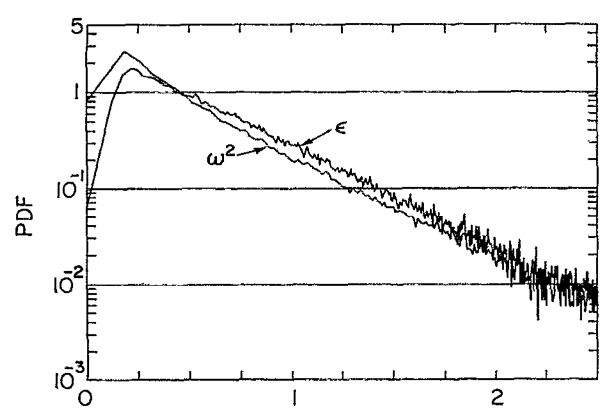

FIG. 2. Probability distribution functions of ω^2 and ϵ close to the jet edge.

It is interesting that in both flows $\gamma_1\simeq 5.7$ while $\gamma_2\simeq 7.2$ for the grid flow ($Re_\lambda=74$) and $\gamma_2=6.3$ for the jet center ($Re_\lambda=880$). Our conjecture is that γ_2 is decreasing with increasing Reynolds number and becomes equal to γ_1 at very large Re. However, this does not mean that the enstrophy generation vanishes as the Reynolds number increases since, for instance, $\langle\sigma^2\rangle^{1/2}$ can increase to infinity. Again it is quite possible that $\gamma_1\simeq 5.7$ (and $\gamma_2\rightarrow\gamma_1$, at large enough Re) is close to some universal quantity. Another invariant $s_{ij}s_{jk}s_{ki}$ exhibits a behavior similar to that of the enstrophy generating term $\sigma=\omega_i\omega_j s_{ij}$.

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